
Price-to-Earnings Ratios: Growth and Discount Rates

Andrew Ang

*Ann F. Kaplan Professor of Business
Columbia University*

Xiaoyan Zhang

*Associate Professor of Finance
Krannert School of Management, Purdue University*

In a present-value model, movements in price-to-earnings ratios must reflect variations in discount rates (which embed risk premiums) and growth opportunities (which involve the cash flow and earnings-generating capacity of the firm's investments).¹ We decomposed P/Es into a no-growth value, defined to be the perpetuity value of future earnings that are held constant with full payout of earnings, and the present value of growth opportunities (PVGO), which is the value of the stock in excess of the no-growth value. To accomplish this decomposition, we used a dynamic model that accounts for time-varying risk premiums and stochastic growth opportunities.

An important aspect of our work is that we took into account a stochastic investment opportunity set with time-varying growth and discount rates. P/Es can be high not only when growth opportunities are perceived to be favorable but also when expected returns are low. For example, during the late 1990s and early 2000s, P/Es were very high. The cause might have been high prices incorporating large growth opportunities, but Jagannathan, McGrattan, and Scherbina (2000) and Claus and Thomas (2001), among others, have argued that during this time, discount rates were low. In contrast to our no-growth and PVGO decompositions, in which both discount rates and growth rates are stochastic, in the standard decompositions of no-growth and PVGO components, discount rates and growth rates are constant. Other standard analyses in the industry, such as the ratio of the P/E to growth (often called the "PEG ratio"), implicitly assign all variations in P/Es to growth opportunities because the analyses do not allow for time-varying discount rates.

¹This approach decomposes the value of a firm into the value of its assets in place plus real options (or growth opportunities). This decomposition was recognized as early as Miller and Modigliani (1961).

Static Case

An instructive approach is to consider first the standard decomposition of the P/E into the no-growth and growth components that is typically done in an MBA-level finance class. The exposition here is adapted from Bodie, Kane, and Marcus (2009, p. 597).

Suppose earnings grow at rate g , the discount rate is δ , and the payout ratio is denoted by po . The value of equity, P , is then given by

$$P = \frac{EA \times po}{\delta - g}, \quad (1)$$

where EA is expected earnings next year. The P/E—that is, $P/E = P/EA$ —is then simply

$$\begin{aligned} P/E &= \frac{P}{EA} \\ &= \frac{po}{\delta - g}. \end{aligned} \quad (2)$$

We can decompose market value P into a no-growth component and a growth component. The growth component is considered to be the PVGO. The no-growth value, P^{ng} , is defined as the present value of future earnings with no growth (so, $g = 0$ and $po = 1$):

$$P^{ng} = \frac{EA}{\delta}. \quad (3)$$

The growth component is defined as the remainder:

$$\begin{aligned} PVGO &= \frac{EA \times po}{\delta - g} - \frac{EA}{\delta} \\ &= \frac{EA[g - (1 - po)\delta]}{\delta(\delta - g)}, \end{aligned} \quad (4)$$

and the two sum up to the total market value:

$$P = P^{ng} + PVGO. \quad (5)$$

The decomposition of firm value into no-growth and PVGO components is important because, by definition, the no-growth component involves only discount rates whereas the PVGO component involves both the discount rate and the effects of cash flow growth. Understanding which component dominates gives insight into what drives P/Es. The static case cannot be used to decompose P/Es into no-growth and PVGO values over time, however, because it assumes that earnings growth (g), discount rates (δ), and payout ratios (po) remain

constant over time. Clearly, this assumption is not true. Thus, to examine the no-growth and PVGO values of P/Es, we need to build a dynamic model.

The Dynamic Model

We made two changes to the static case to handle time-varying investment opportunities. First, we put “ t ” subscripts on the variables to indicate that they change over time. Second, for analytical tractability, we worked in log returns, log growth rates, and log payout ratios.

We defined the discount rate, δ_t , as

$$\delta_t = \ln E_t \left(\frac{P_{t+1} + D_{t+1}}{P_t} \right), \tag{6}$$

where P_t is the equity price at time t and D_t is the dividend at time t . Earnings growth is defined as

$$g_t = \ln \left(\frac{EA_t}{EA_{t-1}} \right), \tag{7}$$

where EA_t is earnings at time t . Finally, the log payout ratio at time t is

$$po_t = \ln \left(\frac{D_t}{EA_t} \right). \tag{8}$$

In this notation, if $\delta_t = \bar{\delta}$, $g_t = \bar{g}$, and $po_t = \bar{po}$ are all constant, then the familiar P/E in Equation 2 written in simple growth rates or returns becomes

$$\frac{P}{EA} = \frac{\exp(\bar{po})}{\exp(\bar{\delta} - \bar{g}) - 1}. \tag{9}$$

Factors. We specified factors X_t that drive P/Es. The first three factors in X_t are the risk-free rate, r_t^f ; the earnings growth rate, g_t ; and the payout ratio, po_t . We included two other variables that predict returns: the growth rate of industrial production, ip_t , and term spreads, $term_t$. We selected these variables after considering variables that, on their own, forecast total returns, earnings growth, or both. We also included a latent factor, f_t , that captures variation in expected returns not accounted for by the observable factors. We specified latent factor f_t to be orthogonal to the other factors. Thus, $X_t = (r_t^f \ g_t \ po_t \ ip_t \ term_t \ f_t)'$.

We assumed that state variables X_t follow a vector autoregression (VAR) with one lag:

$$X_{t+1} = \mu + \Phi X_t + \Sigma \varepsilon_{t+1}, \tag{10}$$

where ε_t follows a standard normal distribution with zero mean and unit standard deviation. The companion form, Φ , allows earnings growth and payout ratios to be predictable by both past earnings growth and payout ratios and other macro variables.

The long-run risk model of Bansal and Yaron (2004) incorporates a highly persistent factor in the conditional mean of cash flows. Our model accomplishes the same effect by including persistent variables in X_t , especially the risk-free rate and payout ratio, which are both highly autocorrelated.

To complete the model, we assumed that discount rates δ_t are a linear function of state variables X_t :

$$\delta_t = \delta_0 + \delta_1' X_t. \quad (11)$$

Equation 11 subsumes the special cases of constant total expected returns by setting $\delta_1 = 0$ and subsumes the general case of time-varying discount rates when $\delta_1 \neq 0$. Because f_t is latent, we placed a unit coefficient in δ_1 that corresponds to f_t for identification.

The Dynamic P/E. Under the assumptions shown in Equation 10 and Equation 11, the dynamic P/E can be written as

$$P/E_t = \sum_{i=1}^{\infty} \exp(a_i + b_i' X_t). \quad (12)$$

The coefficients a_i and b_i are given in Appendix A.²

Our model of the P/E belongs to the asset-pricing literature that builds dynamic valuation models. The approaches of Campbell and Shiller (1988) and Vuolteenaho (2002) to model the price/dividend ratio (P/D) and the P/E, respectively, require log-linearization assumptions. In contrast, our model produces analytically tractable solutions for P/Es. Recently, Bekaert, Engstrom, and Grenadier (2010) and van Binsbergen and Koijen (2010) examined dynamic P/Ds, but not P/Es, in models with closed-form solutions. Our model is more closely related to the analytical dynamic earnings models of Ang and Liu (2001) and Bakshi and Chen (2005), in which cash flows are predictable and discount rates vary over time. Ang and Liu, however, modeled price-to-book ratios instead of P/Es, and Bakshi and Chen's model of the P/E requires the payout ratio to be constant.

Growth and No-Growth Components. The no-growth P/E can be interpreted as a perpetuity, where at each time, a unit cash flow is discounted by the cumulated market discount rates prevailing up until that time. In the full P/E in Equation 12, growth occurs by plowing earnings back into the firm. In the no-growth P/E, earnings are fully paid out; consequently, the payout ratio

²A full derivation is available in the online appendix at www.columbia.edu/~aa610.

does not directly influence the no-growth P/E value. The payout ratio is relevant in the no-growth P/E, however, because the payout ratio is a state variable and its dynamics are allowed to influence future earnings through the VAR process.

The no-growth P/E, P/E_t^{ng} , where earnings growth is everywhere 0 and the payout ratio is equal to 1, can be written as

$$P/E_t^{ng} = \sum_{i=1}^{\infty} \exp\left(a_i^* + b_i^* X_t\right), \quad (13)$$

where a_i^* and b_i^* are given in Appendix A.

The present value of growth opportunities is defined as the difference between the P/E, which incorporates growth, and the no-growth P/E:

$$P/E_t = P/E_t^{ng} + PVGO_t. \quad (14)$$

Empirical Results

We used data on dividend yields, P/Es, price returns (capital gains only), and total returns (capital gains and dividends) on the S&P 500 Index from the first quarter (Q1) of 1953 to the fourth quarter (Q4) of 2009.

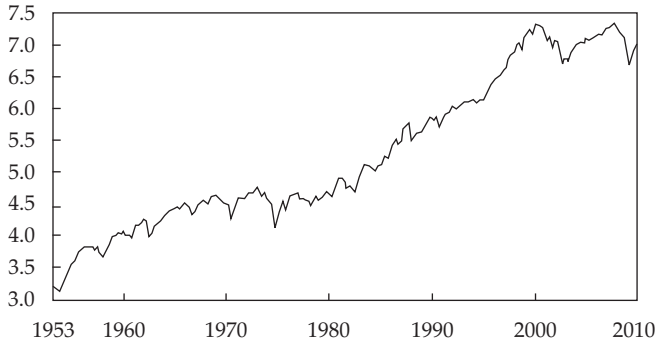
Panel A of **Figure 1** plots the log index of the S&P 500 Total Return Index across our sample. The decline during the mid-1970s recession, the strong bull market of the 1990s, the decline after the technology bubble in the early 2000s, and the drop resulting from the 2008–09 financial crisis are clearly visible. Panel B graphs the P/E, which averages 18.5 over the sample period. The P/E suddenly increased in Q4:2008 to 60.7 and reached a peak of 122 in Q2:2009. In Q4:2009, the P/E came down to 21.9. The large increase in the P/E from Q4:2008 through Q3:2009 is the result of large, negative reported earnings in Q4:2008 during the financial crisis. This development caused the moving four-quarter average of earnings to sharply decrease. While prices were declining during the financial crisis, an even greater decrease was occurring in reported earnings, which caused the increase in the P/E. Panel C of **Figure 1** reports S&P 500 dividend yields, which reached a low at the end of the bull market in 2000.

Estimation Results. **Table 1** reports the parameter estimates of the model. The two most significant predictors of the discount rate are earnings growth, g , with a coefficient of 0.38, and the growth rate of industrial production, ip , with a coefficient of -1.28 . The estimated VAR parameters show that all factors are highly persistent, and this persistence dominates: No other factor except the variables themselves Granger-causes risk-free rates, earnings growth, or payout ratios.³

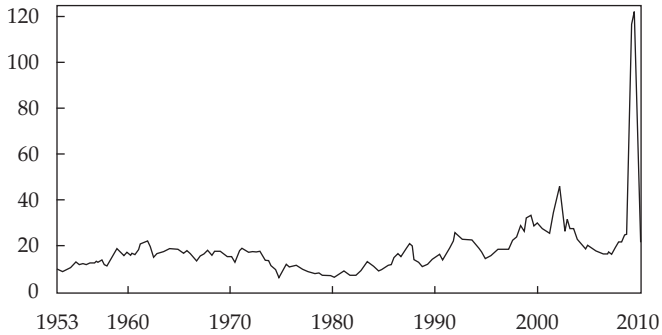
³Estimation of the model is discussed in the online appendix at www.columbia.edu/~aa610.

Figure 1. Log Index Levels, Payout Ratios, and Dividend Yields for S&P 500 Total Return Index, Q1:1953–Q4:2009

A. Log of the Index Level



B. P/E



C. Dividend Yield

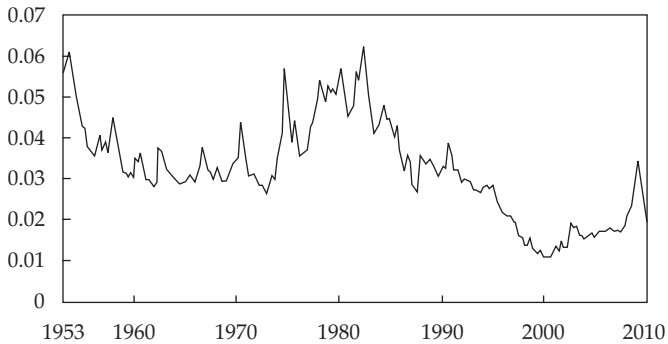
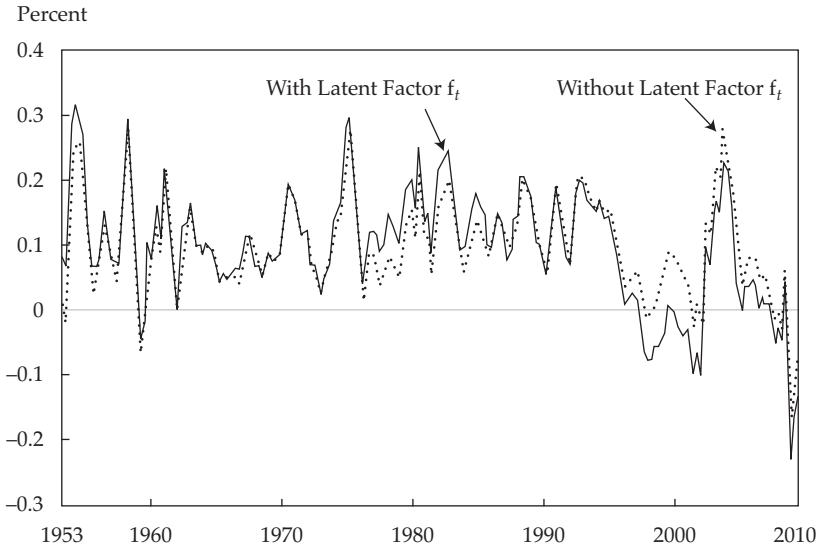


Table 1. Parameter Estimates
(*p*-values in parentheses)

	<i>r^f</i>	<i>g</i>	<i>p_o</i>	<i>i_p</i>	<i>term</i>	<i>f</i>
Discount rate parameters δ_1 :	0.325 (0.775)	0.381 (0.121)	0.164 (0.088)	-1.283 (0.238)	1.203 (1.728)	1 —
VAR parameter Φ						
<i>r^f</i>	0.863 (0.089)	0.26 (0.008)	0.012 (0.012)	-0.005 (0.033)	0.088 (0.191)	0 —
<i>g</i>	0.917 (1.385)	0.628 (0.353)	0.650 (0.426)	0.115 (0.362)	3.677 (3.446)	0 —
<i>p_o</i>	-0.771 (1.292)	-0.514 (0.328)	0.303 (0.415)	0.045 (0.360)	-2.805 (3.131)	0 —
<i>i_p</i>	-0.244 (0.237)	0.096 (0.057)	0.071 (0.041)	-0.169 (0.108)	0.908 (0.737)	0 —
<i>term</i>	0.021 (0.036)	-0.017 (0.005)	-0.003 (0.007)	-0.025 (0.019)	0.502 (0.092)	0 —
<i>f</i>	0 —	0 —	0 —	0 —	0 —	0.904 (0.003)

We plotted the estimated discount rates in **Figure 2**. The full discount rate (solid line) is overlaid with the implied discount rate without the latent factor, f_t (dotted line). The two discount rates have a correlation of 0.91. Thus, the observable factors capture most of the variation in expected returns. Without the latent factor, the observable factors $z_t = (r_t^f, g_t, p_{o,t}, i_{p,t}, term_t)$ account for 18.0 percent of the variance of total returns; adding the latent factor brings the proportion up to 27.5 percent.

Figure 2 shows that discount rates declined noticeably in the 1990s—from 14.5 percent in Q1:1991 to -14.5 percent in Q1:2002. The -14.5 percent corresponds to what was at that time the all-time-high P/E in the sample, 46.5. The latent factor was very negative during this time; the model explains the high P/E as coming from low discount rates. Recently, during the financial crisis, discount rates were again negative. For example, in Q4:2008, the discount rate was -16.3 percent. Q4:2008 was characterized by pronounced negative reported earnings. The P/E increased to 60.7 at this time because of the low earnings relative to market values. The model again explains the high P/E by the low discount rate. The low discount rates at this time were caused by the large decrease in earnings growth. Subsequent returns over the 2008–09 period were indeed extremely low.

Figure 2. Discount Rates, Q1:1953–Q4:2009

Drivers of the P/E. In Table 2, we report variance decompositions of the P/E. We computed the variance of the P/E implied by the model through the sample, where the factor z was held constant at its unconditional mean, $\text{var}_z(P/E)$. The variance decomposition resulting from factor z is given by $1 - \text{var}_z(P/E)/\text{var}(P/E)$, where $\text{var}(P/E)$ is the variance of the P/E in the data. These decompositions do not sum to 1.0 because the factors are correlated. Table 2 shows that the macro variables play a large role in explaining the dynamics of P/Es. Risk-free rates, earnings growth, and payout ratios explain, respectively, 18 percent, 38 percent, and 66 percent of the variance of P/Es.

Table 2. Variance Decompositions of the P/E

Parameter	Variance Explained
rf	17.8%
g	38.3
po	65.9
ip	-38.6
$term$	7.5
f	70.5

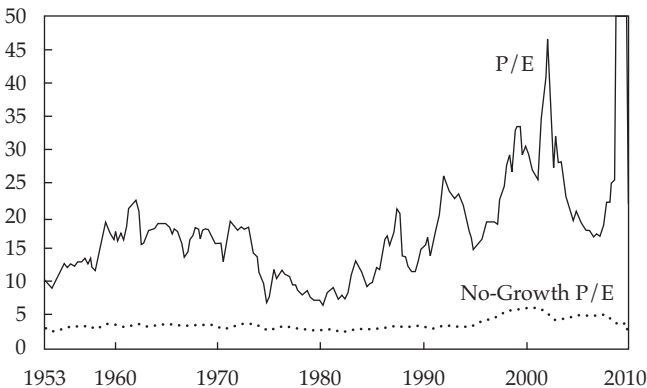
The variance attribution for growth in industrial production is negative because diminished industrial production results in more volatile discount rates and greater volatility of P/Es. The latent factor, f , plays an important role in matching P/Es, with a variance attribution of 71 percent. This finding is consistent with Figure 2, where some occasionally pronounced differences are visible between discount rates produced only with macro variables and discount rates estimated with the latent factor.

Growth and No-Growth Decompositions. Figure 3 plots the no-growth components together with the P/E. Most of the variation in the P/E is a result of growth components. The average no-growth P/E defined in Equation 13 is 3.8, compared with an average P/E in the data of 18.5. Thus, no-growth components account for, on average, 20.7 percent of the P/E; most of the total P/E is a result of the PVGO. The no-growth component is remarkably constant (as is clearly shown in Figure 3) and has a volatility of 0.853, compared with a volatility of 12.7 for the P/E. A variance decomposition of the P/E is

$$\begin{matrix} \text{var}(P/E_t) = & \text{var}(P/E_t^{ng}) & + & \text{var}(PVGO_t) & + & 2\text{cov}(P/E_t^{ng}, PVGO_t) \\ \text{100\%} & \text{0.5\%} & & \text{94.8\%} & & \text{4.7\%} \end{matrix} \quad (15)$$

Thus, 95 percent of P/E variation is explained by growth components, or the PVGO term. The perpetuity value of no-growth is relatively constant because discount rates are highly mean reverting: The year-on-year autocorrelation of discount rates over the sample is 0.34. Thus, the discounted earnings in the no-growth P/E rapidly revert to their long-term average.

Figure 3. No-Growth and Growth Components of the P/E, Q1:1953–Q4:2009



In **Table 3**, we report various correlations of the no-growth and PVGO P/Es. The no-growth and PVGO components have a correlation of 0.363, but this correlation has only a small effect on total P/E variation because of the low volatility of no-growth P/E values. Thus, most of the variation in the total P/E is caused by growth opportunities, and not surprisingly, the PVGO P/E and the total P/E are highly correlated, at 0.998. Both the growth P/E and the total P/E decrease when risk-free rates and earnings growth increase. The correlation of the total P/E with earnings growth is particularly strong at -0.766 . High earnings growth by itself increases earnings, which is the denominator of the P/E, and causes P/Es to decrease, resulting in the high negative correlation between earnings growth and the P/E. But another discount rate effect occurs because high earnings growth causes discount rates to significantly increase (see Table 1). This effect also causes P/Es to decrease. High payout ratios, as expected, are positively correlated with the P/E at 0.713. Finally, the latent factor, f , is negatively correlated with the P/E because it is only a discount rate factor: By construction, P/Es are high when f is low.

Table 3. Correlation of Growth (PVGO) and No-Growth Components of the P/E

	No Growth P/E	PVGO P/E
PVGO P/E:	0.363	
Data P/E:	0.421	0.998
rf	-0.353	-0.426
g	-0.051	-0.766
po	-0.292	0.713
ip	0.114	-0.303
$term$	0.027	0.390
f	-0.903	-0.538

Conclusion

We decomposed the P/E into a no-growth component (the perpetuity value of future earnings held constant with full payout) and a component termed PVGO that reflects the growth opportunities and real options a firm has to invest in the future. We valued both components in a dynamic stochastic environment where risk premiums and earnings growth are stochastic. We found that discount rates exhibit significant variation: 27.5 percent of the variation in total returns is caused by persistent, time-varying expected return components. However, although the variation of discount rates is large, these rates are highly

mean reverting. The result is that the no-growth value of earnings exhibits relatively little volatility. The PVGO component dominates; it accounts for the bulk of the level and variation of P/Es in the data: Approximately 80 percent of the level and 95 percent of the variance of P/Es are a result of time-varying growth opportunities.

We thank Geert Bekaert, Sigbjørn Berg, and Tørres Trovik for helpful discussions.

Appendix A

Here, we provide the coefficients a_i and b_i and the definition of the P/E as used by the S&P 500. All the formulas are derived in the online appendix at www.columbia.edu/~aa610.

Full and No-Growth P/Es. The coefficients a_i and b_i for the P/E in Equation 12 are given by

$$a_{i+1} = -\delta_0 + a_i + (\mathbf{e}_2 + \mathbf{b}_i)' \boldsymbol{\mu} + \frac{1}{2} (\mathbf{e}_2 + \mathbf{b}_i)' \Sigma \Sigma' (\mathbf{e}_2 + \mathbf{b}_i)$$

and

$$b_i = -\boldsymbol{\delta}_1 + \Phi'(\mathbf{e}_2 + \mathbf{b}_i),$$

where \mathbf{e}_n is a vector of 0s with a 1 in the n th position. The initial conditions are

$$a_1 = -\delta_0 + (\mathbf{e}_2 + \mathbf{e}_3)' \boldsymbol{\mu} + \frac{1}{2} (\mathbf{e}_2 + \mathbf{e}_3)' \Sigma \Sigma' (\mathbf{e}_2 + \mathbf{e}_3)$$

and

$$b_1 = -\boldsymbol{\delta}_1 + \Phi'(\mathbf{e}_2 + \mathbf{e}_3).$$

The coefficients in the no-growth P/E, P/E_i^{ng} , in Equation 13 are given by

$$a_{i+1}^* = -\delta_0 + a_i^* + \mathbf{b}_i^{*'} \boldsymbol{\mu} + \frac{1}{2} \mathbf{b}_i^{*'} \Sigma \Sigma' \mathbf{b}_i^*$$

and

$$\mathbf{b}_{i+1}^* = -\boldsymbol{\delta}_1 + \Phi' \mathbf{b}_i^*,$$

where a_i^* and \mathbf{b}_i^* have initial values $a_i^* = -\delta_0$ and $\mathbf{b}_i^* = -\boldsymbol{\delta}_1$.

Data. The P/E defined by Standard & Poor's is the market value at time t divided by trailing 12-month earnings reported from t to $t - 1$. To back out earnings growth from P/E's, we used the following transformation:

$$\begin{aligned}\exp(g_{t+1}) &= \frac{EA_{t+1}}{EA_t} \\ &= \left(\frac{P/E_t}{P/E_{t+1}} \right) \left(\frac{P_{t+1}}{P_t} \right),\end{aligned}$$

where P_{t+1}/P_t is the price gain (capital gain) on the market from t to $t + 1$.

The dividend yield reported by Standard & Poor's is also constructed from trailing 12-month summed dividends. We computed the log payout ratio from the ratio of the dividend yield, $dy_t = D_t/P_t$, to the inverse P/E:

$$\begin{aligned}\exp(po_t) &= \frac{dy_t}{1/(P/E)_t} \\ &= \frac{D_t}{EA_t}.\end{aligned}$$

For the risk-free rate, r_t^f , we used one-year zero-coupon yields expressed as a log return, which we obtained from the Fama Files derived from the CRSP U.S. Government Bond Files. For the macro variables, we expressed industrial production growth, ip , as a log year-on-year growth rate for which we used the industrial production index from the St. Louis Federal Reserve. We defined the term spread, $term$, as the difference in annual yields between 10-year and 1-year government bonds, which we obtained from CRSP.

BIBLIOGRAPHY

- Ang, A., and J. Liu. 2001. "A General Affine Earnings Valuation Model." *Review of Accounting Studies*, vol. 6, no. 4 (December):397–425.
- Bakshi, G., and Z. Chen. 2005. "Stock Valuation in Dynamic Economies." *Journal of Financial Markets*, vol. 8, no. 2 (May):115–151.
- Bansal, R., and A. Yaron. 2004. "Risk for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance*, vol. 59, no. 4 (August):1481–1509.
- Bekaert, G., E. Engstrom, and S.R. Grenadier. 2010. "Stock and Bond Returns with Moody Investors." *Journal of Empirical Finance*, vol. 17, no. 5 (December):867–894.
- Bodie, Z., A. Kane, and A.J. Marcus. 2009. *Investments*. 8th ed. New York: McGraw-Hill/Irwin.
- Campbell, J.Y., and R.J. Shiller. 1988. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies*, vol. 1, no. 3 (July):195–228.

Claus, J., and J. Thomas. 2001. "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets." *Journal of Finance*, vol. 56, no. 5 (October):1629–1666.

Fama, E.F., and K.R. French. 2002. "The Equity Premium." *Journal of Finance*, vol. 57, no. 2 (April):637–659.

Jagannathan, R., E.R. McGrattan, and A. Scherbina. 2000. "The Declining U.S. Equity Premium." *Federal Reserve Bank of Minneapolis Quarterly Review*, vol. 24, no. 4 (Fall):3–19.

Miller, M.H., and F. Modigliani. 1961. "Dividend Policy, Growth, and the Valuation of Shares." *Journal of Business*, vol. 34, no. 4 (October):411–433.

van Binsbergen, J., and R.S.J. Koijen. 2010. "Predictive Regressions: A Present-Value Approach." *Journal of Finance*, vol. 65, no. 4 (August):1439–1471.

Vuolteenaho, T. 2002. "What Drives Firm-Level Stock Returns?" *Journal of Finance*, vol. 51, no. 1 (February):233–264.