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Investing Separately in Alpha and Beta

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Foreword

Some innovations spread quickly. The web browser is a case in point. Others are adopted more slowly. Between 1952, when Harry Markowitz showed how to factor both the risks and the expected returns of securities into a portfolio construction decision, and 1964, when William Sharpe published the best-known rendition of the capital asset pricing model, the idea that the returns on an asset (any asset) consist of a market part and a nonmarket part came to fruition.¹ The market part, now called “beta,” is the part of the return that is explained by correlation with one or more broad-based market indices. The part of the return not explained by beta is the “alpha,” usually interpreted as the return from active management skill. This idea was solidified in a 1967 article by Michael Jensen, and the meanings of alpha and beta have changed little since then.² Thus, alpha and beta have been clearly separate—as concepts—for about 40 years.

Within less than a decade after Jensen’s work, the concepts of alpha and beta began to be used in performance measurement and in setting incentive fees.³ Although managers chafed at having their performance measured, customers and their consultants insisted that managers justify their active fees by performing better than a comparable index fund. The retrospective measurement of alpha and beta for stock portfolios and, ultimately, for portfolios in other asset classes became almost universal practice.

Yet, *investing* separately in alpha and beta, which one might think an easy extrapolation from measuring alpha and beta as separate quantities, is a relatively recent phenomenon, dating back only to the 1990s. The basic way to invest separately in alpha and beta is to purchase two funds: a market-neutral, zero-beta portfolio to earn “pure” alpha and another fund (which may or may not be in the same asset class as the first one) to add desired beta exposures. The authors of this monograph—Roger Clarke, Harindra de Silva, and Steven Thorley—take this classic portable alpha design as their starting point but not their endpoint.

To build the classic portable alpha strategy, one must have access to the needed investment vehicles. The burgeoning growth of the hedge fund marketplace in the 1990s and in the first decade of this century produced a supply of market-neutral

¹Harry M. Markowitz, “Portfolio Selection,” *Journal of Finance*, vol. 7, no. 1 (March 1952):77–91; William F. Sharpe, “Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk,” *Journal of Finance*, vol. 19, no. 3 (September 1964):425–442.

²Michael C. Jensen, “The Performance of Mutual Funds in the Period 1945–1964,” *Journal of Finance*, vol. 23, no. 2 (May 1967):389–416.

³Laurence B. Siegel, *Benchmarks and Investment Management* (Charlottesville, VA: Research Foundation of the Association for Investment Management and Research, 2003):31.

hedge funds that provided alpha without beta. (Although most hedge funds are not market neutral and thus expose the investor to various betas, as well as alpha opportunities, those hedge funds that are market neutral form the natural basis for a portable alpha strategy.)

In addition, one needs a cheap and efficient beta source. Because of the usual budget constraint that one cannot invest more than 100 percent of cash on hand, the beta source cannot be a conventional index fund; rather, it must be sought in the futures or swap market, where margin requirements are minimal. Thus, the creation of a liquid market for derivatives on various asset class indices, which began in the 1970s, was a precondition for the emergence of portable alpha as a viable strategy.

Of course, investing separately in alpha and beta involves a kind of leverage. Although the cash required by the strategy typically does not exceed 100 percent of the investor's available funds, the resulting exposures do sum to more than 100 percent. This "economic leverage," however, may not be "recourse leverage" in the sense of investing borrowed funds that must be paid back irrespective of the investment result.

As the investment manager Howard Marks has said (in a memo to clients), "Volatility + Leverage = Dynamite."⁴ When markets turn sour, the returns of a leveraged strategy can be catastrophic while those of an unleveraged strategy are merely disappointing. In the crash of 2008, some portable alpha strategies reported a return of -60 percent, consisting of a beta, or market return, of -40 percent "ported" on top of an alpha of -20 percent. This result suggests poor execution of the alpha part of the strategy and in no way invalidates portable alpha as a concept. A pure alpha of -20 percent is extremely unusual and suggests that the supposedly market-neutral managers had hidden beta exposures. The lessons of this episode are twofold: One must always be on guard against the masquerading of beta as alpha when selecting alpha managers, and one must remain mindful that alpha is as likely to be a negative number as it is a positive number.

Clarke, De Silva, and Thorley's monograph is not limited to a discussion of portable alpha. Another strategy that they consider in detail is the "reunion of alpha and beta." The intellectual underpinning of alpha-beta separation is the idea that one can add value by removing a number of expensive constraints that are present in traditional portfolios. These constraints include the no-shorting constraint, the no-leverage or budget constraint (i.e., portfolios can be no more than 100 percent invested), and the constraint that alpha and beta must come from the same asset class. But as my discussion of the 2008 crash suggests, unconstrained portfolios may be too risky for some investors. One solution is to put back some, though not all, of the constraints. This is accomplished through such structures as the 130/30 fund

⁴Howard Marks, "Volatility + Leverage = Dynamite" (Los Angeles: Oaktree Capital Management, 2008).

(130 percent long and 30 percent short), which puts back three constraints: (1) Alpha and beta are sourced from the same asset class (the “reunion”), (2) the beta is equal to 1, and (3) gross exposure—long plus short positions as measured by their absolute value—does not exceed 160 percent. These constraints limit risk while preserving the advantage of being able to sell overpriced securities short.

The authors make this tutorial monograph come alive by using case studies from the world of pension fund management, where portable alpha and related strategies have been widely adopted. They have put great effort into presenting detailed examples that can make the difference between superficial understanding and deep comprehension. We are delighted to present this practical users’ guide to investing separately in alpha and beta.

Laurence B. Siegel
Research Director
Research Foundation of CFA Institute

1. Introduction

The Greek letters “alpha” and “beta” are popping up everywhere in investment management practice. Like option market participants with their “delta” and “gamma” and risk managers with their “sigma” (lately, multiple-sigma) events, alpha and beta have become standard vernacular among investment managers, consultants, and plan sponsors. Even fiduciary boards and investment committees are speaking Greek. Alpha, once a technical term associated with performance measurement, is being ported, attached, marked up, earned, and occasionally lost. Meanwhile, betas are being hedged, replicated, commodified, and happily reunited with alphas, although not always with the same alpha that brought them to the dance.

The separation of alpha and beta sources of return in institutional portfolios has arrived and is having a profound influence on the way investors view risk and return. Some observers believe that the impact of alpha–beta separation will be as transformative as modern portfolio theory was in the 1960s, while others consider it merely a passing fad. As usual, the truth is probably somewhere in the middle, but the need for a better understanding of alpha–beta principles and terminology among investment professionals is clear. The market turmoil of 2008 has stressed most institutional portfolios, regardless of whether they were constructed with an eye toward alpha–beta separation. The goal of this monograph is to provide an objective source of information on alpha–beta separation for the institutional investment community—particularly pension plan sponsors, foundations, and endowments—so that using the concepts does not create false expectations for investors. A small avalanche of white papers, journal articles, books, and other sources of information on alpha–beta separation has recently become available from a variety of sources. We hope that this monograph collects the important content in one place for professionals who need access to alpha–beta principles, terminology, current practice, and implementation issues.

Some caveats are in order. First, several different conceptual frameworks are associated with the word “beta” in asset management, including the original capital asset pricing model (CAPM) from financial economics. As explained in Chapter 2, we do not intend to resolve any of the outstanding academic debates about what constitutes true beta; instead, we generally use the term in the practical sense of any market exposure that can be cheaply replicated. Second, we mention some investment management firms and funds by name in our discussion of alpha–beta separation—particularly in the empirical and applications chapters. We hope this approach gives more color and real-world orientation to the monograph, but we do not endorse these particular investment management firms over any other. We

encourage readers to pursue standard search and due diligence procedures in the process of evaluating potential investment managers and products. Third, the presentation in this monograph assumes a familiarity with standard investment principles and terminology at a level expected of a CFA charterholder or an investment professional with several years of experience. Although we define terms specific to alpha–beta separation, general portfolio management concepts and vocabulary are used without detailed explanation. The monograph relies on the underlying principles of standard portfolio theory, particularly in Chapter 3, but we generally relegate equations to the Appendices (A–D).

The concepts related to alpha–beta separation are numerous and subtle enough to fill this entire monograph, but the main idea can be expressed as follows: Traditionally, institutional investors have approached portfolio structure in two stages. First, they establish the policy portfolio or allocation to various asset classes (the beta stage); second, they choose active and passive managers to implement the allocation within each asset class (the alpha stage). This traditional approach naturally attaches the potential added value of active managers to the asset class in which the active management takes place.

Increasingly, institutional portfolios are being built by considering active (alpha) returns separately from broad market (beta) returns. Versions of this conceptual framework have been used for many years in the context of *ex post* performance attribution and, more recently, by some institutions in the process of *ex ante* risk budgeting. What is new is the advent and wide acceptance of shorting and derivative securities—specifically, financial market futures and exchange-traded funds (ETFs)—in institutional portfolio practice. The use of derivative securities to hedge and replicate market risk means that value added through active management need not be tied to the asset class in which the active management takes place. The literal, rather than merely conceptual, division of active return exposure and broad market return exposure into separate products gives plan sponsors and other institutions new flexibility in portfolio construction.

For example, an institution that decides to maintain a large allocation to domestic equity can do so with or without any attempt to seek alpha from domestic equity. Alternatively, an institution that believes it has access to a fund manager who can produce alpha in some less prominent asset category may go after that alpha with or without any commitment to the asset class itself. Alpha is separable, portable, and fungible; it does not really matter where the alpha comes from (equity alpha is the same as fixed-income alpha, which is the same as global tactical asset allocation [GTAA] alpha), and more alpha is better than less alpha. Investors are free to establish the portfolio’s market exposure on the basis of market risks and returns while seeking a portfolio of alpha sources wherever and whenever they can be found. One can liken the alpha–beta separation principle to the designated hitter position in baseball. The batting prowess of one player can be separated from the fielding or pitching prowess of another so that the team gets the best of both.

In Chapter 2, we continue our discussion of this central idea, together with several others, including historical background, conceptual frameworks, and terminology. Chapter 2 emphasizes that alpha is rare and expensive and beta is ubiquitous and relatively cheap. “Portable alpha” is only one manifestation of a broader alpha–beta separation framework. The use of derivative securities in beta replication and hedging requires an appreciation of contingent versus actual capital commitments. Chapter 3 demonstrates that for traditional active funds, the whole is actually worth *less* than the parts. Portfolio theory shows that the separation of a fund into its alpha and beta sources of return leads to an improved risk–return trade-off. We also explain how alpha–beta separation naturally leads to a total portfolio risk-budgeting process. Chapter 4 provides several empirical examples of alpha–beta separation in the familiar equity and fixed-income asset classes, a discussion of the beta factors implicit in those asset classes, and an empirical analysis of hedge funds. Chapter 5 consists of five “case studies” indicating how plan sponsors are adapting to and using alpha–beta separation principles in their portfolios and includes comments on common practices. Chapter 6 reviews implementation issues, including the search for alpha, synthetic beta management, and the econometrics of beta (and, by extension, alpha) measurement. Chapter 7 covers a related development in portfolio management theory: the reunion of alpha and beta in the context of long–short extension (e.g., 130/30) strategies. In Chapter 8, we summarize important concepts and prognosticate about future trends in asset management practice from an alpha–beta separation perspective.

2. Alpha–Beta Separation: History and Concepts

This chapter continues our discussion of the central concepts in alpha–beta separation, including a brief review of the history of ideas that brought separation about and a delineation of various conceptual frameworks for viewing alpha and beta. We explain that our focus will be on alpha and beta separation from the perspective of exposure replication and hedging. We also discuss the concept of contingent versus committed capital and associated terminology.

History and Background

The Greek letters “alpha” and “beta” were first introduced to the asset management industry in the 1960s through the CAPM, which was originally proposed as an equilibrium theory of expected returns under a set of simplifying assumptions about investor preferences and market structure (Treynor 1962; Sharpe 1964; Lintner 1965; Mossin 1966). Debate about the empirical validity of the CAPM as an adequate description of security markets began as soon as the theory was propounded and continues today, but the basic concepts have long since become embedded in asset management vocabulary.

One key CAPM concept is the decomposition of security returns into two parts: a portion attributable to general market movements and an idiosyncratic portion that is not. The simple “market model,” first set forth in Sharpe (1963), specifies a linear relationship between individual stock and market returns over time:

$$(r_i - r_F) = \alpha_i + \beta_i (r_M - r_F) + \varepsilon, \quad (2.1)$$

where r_i is an individual stock return, r_M is the market return, and r_F is the risk-free rate. Following the common notation for linear regressions, the Greek letter beta is the “slope” term in Equation 2.1 and alpha is the “intercept” term. The alpha (α_i) and beta (β_i) in Equation 2.1 are subscripted by “ i ” to designate these parameters as specific to the i th out of N stocks in the market. (Epsilon, ε , is an error term distributed randomly around zero.) Note that both security and market returns in Equation 2.1 are measured in *excess* of the contemporaneous risk-free (e.g., Treasury bill) rate. The CAPM adapts the market model for security returns given in Equation 2.1—and adds a number of simplifying assumptions about investor behavior and market structure—to yield an economic theory of security market equilibrium. The pathbreaking conclusion that eventually earned one of its originators, William Sharpe, the 1990 Nobel Prize in economics is that a security’s expected (and thus average realized) return is solely dependent on its market beta. A summary of the assumptions and implications of the CAPM is provided in Appendix D.

The CAPM was largely ignored by the money management industry until another economist, Michael Jensen, used it to measure the performance of mutual funds in the late 1960s (Jensen 1968). Jensen's alpha, as it came to be known, is the historical average portfolio excess return minus the product of the portfolio's beta and the market excess return over some specific time period, where "excess" means "excess over the riskless rate." In Jensen's application, a portfolio manager earns positive alpha by selecting stocks that subsequently have realized returns higher than those predicted by the CAPM. Ironically, CAPM theory and terminology, which are based on the assumption of informationally efficient markets, began to be used as a basis for measuring the degree to which a manager could exploit market inefficiencies.

Although the CAPM provided the initial terminology and conceptual framework for the separation of alpha and beta, practical separation became more relevant with the emergence of index funds in the 1980s. As soon as an investor accepts low-cost indexing as the default strategy, active portfolio management adds value only by choosing securities that outperform the index. Furthermore, if a manager simply chooses securities that have historically shown a high sensitivity to market movements (a high beta) and the market subsequently goes up, no true value is added even if the portfolio increases by more than the market. Such added exposure to market returns can easily be replicated by leveraging an index fund to give it a higher beta. For example, if the actively managed portfolio is composed of stocks that have an average market beta of 1.2 and the market goes up by 5 percent in excess of the risk-free rate, a 6 percent excess return on the actively managed portfolio has an alpha of zero. Although originally intended as a theoretical measure of systematic risk and expected security returns in financial market equilibrium, beta turned out to have additional usefulness as a measure of the sensitivity of a security or portfolio to the general market, without reference to capital market equilibrium conditions.

At least one other development from financial economics is relevant to our discussion of the separation of alpha and beta. By most accounts, empirical tests of the traditional CAPM have disappointed financial economists. Expected security returns, as inferred from average realized returns over long periods of time, show only a weak relationship with market beta and have significant relationships with a number of factors not included in the CAPM (Fama and MacBeth 1973). In response, financial economists began to develop alternative equilibrium models under the names of multifactor CAPM (Merton 1973) and arbitrage pricing theory (Ross 1976). The expected returns of individual securities (and, by extension, entire portfolios) in these more general asset-pricing theories are a function of *several* risk factors, one of which might or might not be the general market. From this perspective, the original CAPM is a "single-factor" model in which the sole factor is the market return. In economic theory, these multiple factors should represent sources of nondiversifiable risk to qualify as determinants of positive expected return. In practice, the list of potential factors has been largely driven by observed patterns in historical market returns and by the development of derivative securities that attempt to replicate factor exposures.

Like the original CAPM beta, many managers use multifactor betas without weighing in on the issue of a long-term positive payoff predicted by equilibrium theory, and multifactor perspectives now dominate the way economists and most practitioners view financial markets. The “correct” list of factors in any given market (e.g., domestic equity, fixed income) is an ongoing debate, and as explained below, the answer largely depends on the conceptual framework or motivation for specifying factors. For purposes of illustration, we mention here the well-known three-factor Fama–French (1996) model for public equity, which adds “size” and “value” factors to the general market factor in Equation 2.1:

$$(r_i - r_F) = \alpha_i + \beta_i (r_M - r_F) + \beta_{SMB,i} SMB + \beta_{HML,i} HML + \varepsilon. \quad (2.2)$$

The acronym SMB stands for “small minus big” stock returns and was motivated by the empirical observation that over long periods of time, small-capitalization stocks in the United States have had higher returns than large-cap stocks (Banz 1981; Reinganum 1983). Similarly, the acronym HML stands for the return on “high minus low” book-to-market-ratio stocks and is based on the empirical observation that value stocks (those with high book-to-market ratios) tend to outperform growth stocks (Fama and French 1992). Note that Fama and French invert the more familiar valuation ratio of market-to-book. Prior variants of the Fama–French value/growth stock classification use the price-to-earnings ratio (or its inverse, earnings yield) to classify stocks into value and growth categories (Basu 1977). The actual returns to the SMB and HML factors in the Fama–French model are measured by returns to market-neutral long–short portfolios as specified on Kenneth French’s website (mba.tuck.dartmouth.edu/pages/faculty/ken.french).

Conceptual Frameworks for Beta Factors and Alpha

The allowance for multiple factors in a market, as in the Fama–French model, raises the question, why these factors, and why just three? For example, the competition for a fourth factor in U.S. equity securities was, for a time, a two-way race between dividend yield and price momentum. The momentum factor—first documented in academic journals by Jegadeesh and Titman (1993) and further developed by Carhart (1997)—apparently won that race by receiving its own Fama–French acronym, UMD, for “up minus down” stocks. Furthermore, as attention is extended from domestic equity to fixed-income markets (Fama and French 1993) and international markets, additional candidates for relevant factors might include credit spread, term structure, currency, and geographic (country or region) factors, not to mention line-of-business (sector and industry) factors. Expansion of the focus beyond linear factors might include asymmetric option-based returns and returns to market-timing (tactical asset allocation) strategies. In a multifactor world, what are the appropriate criteria for selecting beta factors? We offer several conceptual frameworks for establishing an appropriate list of beta factors for both the U.S. equity market and global financial markets in general.

Framework 1. Financial Economics. As we have discussed, the original concept of alpha–beta separation came from academic economists who were trying to describe a simple model of market equilibrium in the 1960s. In financial economics, beta factors represent nondiversifiable risks that require a positive expected return, or risk premium, as compensation. For example, Fama and French argued that the positive payoff to small-cap and value stocks observed historically in equity markets represents risk premiums—that is, rewards for some unspecified but nondiversifiable risk factor—rather than market inefficiencies. Indeed, in the set of “neoclassical” economic assumptions that spawned the CAPM and other equilibrium multifactor models, market inefficiencies are either nonexistent or small and transient enough to be inconsequential.

Framework 2. Performance Attribution. As mentioned in the historical review, one of the early practical applications of alpha–beta separation was in mutual fund performance attribution. Specifically, in a single-factor analysis of U.S. equity fund returns, beta represents the general market exposure of a fund and alpha represents the added value or unique talent expressed by the portfolio manager’s security selection process. In contrast to the baseline assumptions of neoclassical economics, performance attribution assumes that substantial market inefficiencies exist and are the source of managerial added value. (In an efficient market, where management skill is impossible, some actively managed funds would have positive *ex post* alphas, but these would necessarily be a result of luck.)

The performance attribution framework acknowledges that although the equity market might be inefficient, active management is still a zero-sum game. As Sharpe (1991) observed, the aggregated portfolios of all participants in any given financial market are, by definition, the capitalization-weighted average return of the securities in that market. Participants who earn positive alphas do so at the expense of other participants with negative alphas. Thus, from a performance attribution perspective, beta factors include any marketwide security characteristic that is associated with significant nonzero returns, whether the return is a result of a risk premium or a persistent market inefficiency. For example, performance attribution systems for U.S. equity portfolios might augment the three-factor Fama–French model by adding momentum as a fourth beta exposure to delineate the “true” alpha added by the manager’s choice of individual securities.

Framework 3. Risk Modeling. The basic framework of formal portfolio optimization was established by Markowitz in the 1950s, although the practical application to large-scale portfolios by using electronic databases and computers is of more recent origin. The portfolio optimization process requires a covariance matrix, which includes the expected variances of individual securities and covariances of security pairs. The matrix of estimates is typically based on a multifactor risk model that specifies common factors, or sources, of covariance between securities and individual security exposures, or betas, on those factors. Even

in the absence of formal portfolio optimization, security return covariance matrices are used to monitor the risk of portfolios, whether passively or actively managed. In this risk-modeling framework, beta factors include any material source of common covariance between security returns, independent of whether the “payoff” to that factor is positive, zero, or negative. For example, although neither theoretically nor empirically associated with long-term positive or negative returns, such risk factors as industry or economic sector membership can explain much of the realized return on an individual stock in any given period.

Framework 4. Exposure Replication and Hedging. The concept of alpha–beta separation is also useful for replicating or eliminating (hedging) exposures to markets or common risk factors. The instruments used for replication or hedging include derivative financial securities (e.g., index futures and ETFs) and return swap agreements. Simple, rule-based security selection or market-timing strategies can also be replicated.

Cost saving is the principal reason that exposure replication strategies and securities exist. Investors are reluctant to pay traditional management fees for the “beta” component of a portfolio because such exposure, with its associated risk and returns, can be obtained at a low cost through derivatives. In modern financial markets, passive investments are not limited to traditional equity index funds; they include a variety of instruments that provide exposures to equity market subfactors, bond markets, emerging equity markets, commodities, and currencies, as well as long or short exposure to volatility in various asset categories. But an index’s tradability alone does not necessarily mean it constitutes pure beta exposure. Some newer indices are a mixture of active management and broad market exposure in that they include elements of active strategies. The replication and hedging motivation for distinguishing between alpha and beta is discussed in recent papers by Kung and Pohlman (2004) and Waring and Siegel (2006).

Although some overlap exists among our four conceptual frameworks for alpha–beta separation, our primary focus is exposure replication and hedging. The existence of such low-cost, market-tracking securities as index futures contracts is both motivated by and required for the portable alpha and pure-alpha strategies we describe in subsequent sections. To some extent, however, we also rely on the performance attribution and financial economics perspectives. With respect to performance attribution, we generally think of alpha generation as a zero-sum game in which some investors exploit market inefficiencies at the expense of other investors who are subject to behavioral biases, less sophisticated analytic or processing skills, or some sort of regulatory constraint. In accordance with financial economics, we tend to view beta exposures as having a positive expected return because they represent a premium or reward for bearing meaningful risk. We do not use the risk-modeling perspective discussed earlier except for the well-developed statistical techniques and mathematical notations to measure and monitor portfolio risk.

Index-Weighting Schemes

Empirical research (e.g., Haugen and Baker 1991; Clarke, De Silva, and Thorley 2006) suggests that capitalization-weighted indices in the U.S. equity market may not be mean–variance efficient. Alternative weighting schemes (e.g., equally weighted or fundamentally weighted equity indices; see Arnott, Hsu, and Moore 2005) have different aggregate exposures to market subfactors and can thus perform differently than cap-weighted indices. So long as an investor chooses a rule-based benchmark, alpha can arguably be measured relative to any benchmark index, or “beta.” Even so, we generally regard alternative weighting schemes as a mixture of alpha and beta returns and risk to the extent that they tilt away from market weights. The cap-weighted composite of individual securities is the only portfolio that all investors can hold simultaneously, in accordance with the clearing definition of “the market” in financial economics. In addition, market-cap-weighted indices are self-rebalancing, except for dividends and membership changes, in contrast to all other weighting schemes, which require active rebalancing to the desired index weights because of ordinary stock price changes.⁵ Active rebalancing suggests that active risk, or alpha risk, is being taken and, therefore, that active return is being generated. In any case, most futures and ETFs—essential to market beta replication and hedging—are based on cap-weighted indices or their close cousins, float-weighted indices.

The terms “alpha” and “beta” are now so frequently used in discussions of portfolio management that both words have acquired shades of meaning that vary with specific circumstances. The original financial economic usage is typically confined to academia, but the performance attribution usages are commonplace and conceptually clear, even if the choice of an appropriate benchmark is not. The wider application of the terms alpha and beta in separation strategies has spawned a number of more nuanced usages. For example, Leibowitz (2005) uses the terms to categorize various approaches to portfolio strategy for institutional investors, with colorful analogies to “alpha hunters” and “beta grazers.” Anson (2008) provides an interesting continuum of betas, ranging from “classic beta” to “bulk beta,” with “alternative beta” somewhere in the middle. **Exhibit 2.1** summarizes the characteristics of alpha versus beta returns and risks that are the most critical to our discussion of alpha–beta separation.

Figure 2.1 illustrates the basic building blocks of risk and return from an alpha–beta separation perspective. The portfolio return is divided into three parts: the riskless return, the risk premium from passive beta exposure, and the alpha return from either (1) active management of individual securities or (2) tactical timing of beta exposures. The beta and alpha components of return each contribute to the volatility of the portfolio, but the riskless return does not. Specifically, passive

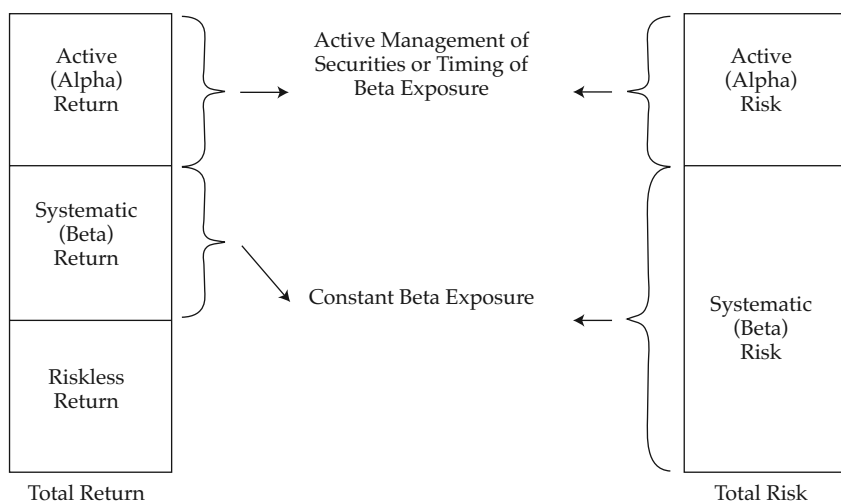
⁵Additional exceptions include corporate actions such as secondary issuance of stock, mergers, acquisitions, and divestitures.

Exhibit 2.1. Characteristics of Alpha and Beta Risks

	Beta Risk	Alpha Risk
Source of return	Positive expected premium earned by passive market exposure over time	Return from actively managing exposures to individual securities or timing of market exposure
Skill required	Low	High—competing with other active managers
Confidence in earning the expected return	High over long periods, but subject to short-term volatility	Low—difficult to identify exceptional managerial talent in advance
Cost	Low	High—have to pay for managerial talent and trading costs
Allocation of return among investors	All investors simultaneously realize the same return for the same market exposure	Some investors earn active returns at the expense of others
Shape of the return distribution	Can have fat tails but is somewhat symmetric	Can be quite skewed (asymmetric distribution), with significant fat-tail risk

beta exposure contributes beta risk and active alpha management contributes active risk. Traditional managers couple beta risk with active risk in the same portfolio in some fixed (and perhaps unintended) proportion. As explained in Clarke, De Silva, and Wander (2002) and as illustrated in Chapter 3 of this monograph, separation provides the investor with flexibility in configuring the amount of risk taken between the two sources.

Figure 2.1. Decomposition of Portfolio Risk and Return



Capital: Committed vs. Contingent

In any given strategy, the actual separation of alpha and beta requires one or more of three different capabilities: short selling, the use of derivatives, and scaling volatility through leverage. The liquid derivatives markets as we know them today did not become well developed until the 1980s. The ability to sell securities short and to use leverage has been used selectively throughout history, but efficient mechanisms for borrowing securities to short-sell and securing leverage credit for institutional portfolio managers have only recently become available. The advent of computing power to communicate information quickly, do complex calculations, assess risk exposures, and track and execute trades has also been critical to the development of strategies that separate alpha and beta.

We can classify alpha and beta sources of return according to whether actual or contingent capital is used for each source. Actual, or *committed*, capital involves the use of cash to purchase securities. *Contingent* capital, a concept introduced by Layard-Liesching (2004), refers to the use of derivatives that require little or no cash up front but that subsequently may require cash to settle losses. The concept of contingent capital encourages the investor to plan for adequate liquidity when losses must be funded. We capture this two-by-two perspective in **Exhibit 2.2**. Beta exposure can be generated by using either securities that require the commitment of actual capital (the upper-left quadrant) or derivatives requiring contingent capital (the upper-right quadrant). A common characteristic of beta exposures is that they are held for an extended period of time and thus generate constant market exposure. Alpha exposure comes from over- and underweighting specific securities within a market, but it can also be generated by tactical short-term shifts in market exposure, or what has been called “active beta” (Kung and Pohlman 2004). In either case, the chief characteristic of alpha generation is the deviation of positions from the benchmark. As shown in the lower half of Exhibit 2.2, alpha from security selection generally requires the commitment of actual capital, whereas alpha from market timing can be achieved with either committed or contingent capital.

Exhibit 2.2. Capital Requirements for Beta and Alpha Exposures

	Committed Capital	Contingent Capital
Beta exposure: risk and return from exposure to market factors	<ul style="list-style-type: none"> • Index funds • ETFs • Structured products 	<ul style="list-style-type: none"> • Futures • Swaps
Alpha exposure: risk and return from active security selection or tactical beta timing	<ul style="list-style-type: none"> • Security selection • Tactical beta allocation 	<ul style="list-style-type: none"> • Tactical beta allocation

Investment Products

With this two-by-two matrix in mind, we can better understand the various products and terminology used in relation to alpha and beta sources of return in investment management. Specifically, we categorize investment products by their position in Exhibit 2.2 as follows:

Index funds (e.g., the well-known S&P 500 fund provided by Vanguard), which provide beta exposure and require the investor to put up actual cash, fall into the upper-left quadrant of Exhibit 2.2.

ETFs (e.g., State Street Corporation's Standard & Poor's Depositary Receipts, or SPDRs ["Spiders"]) are similar to traditional index funds from an alpha-beta perspective and also fall into the upper-left quadrant of Exhibit 2.2.

Synthetic indexing refers to beta exposures obtained through futures or swap contracts and thus falls into the upper-right quadrant of Exhibit 2.2. If the magnitude of the exposure is altered over time in an attempt to time the market, the change in market exposure becomes a source of alpha and thus falls into the lower-right quadrant.

Portable alpha strategies obtain beta exposure through derivatives contracts and leave the actual capital free to fund an unrelated alpha source. Perhaps the best-known product of this kind is PIMCO's Stocks Plus. Stocks Plus uses the fixed-income management skill of PIMCO to generate returns greater than the interest rate embedded in the futures market and then overlays those returns with equity futures contracts to create beta exposure. Portable alpha strategies span the upper-right and lower-left quadrants of Exhibit 2.2.

Enhanced index funds generally refer to funds with very low active risk. The "enhancement" is the attempt to generate alpha through small over- and underweighting of individual securities as compared with the index (e.g., Barclays Global Investors' Alpha Tilts product). Enhanced indexing spans the upper-left and lower-left quadrants of Exhibit 2.2. Some investment professionals also refer to portable alpha strategies as a type of "enhanced indexing."

Diversified beta funds provide exposure to multiple sources of beta—for example, domestic equity, international equity, and fixed income—all within one product. The beta exposures may be generated entirely through funded positions or through a combination of cash and derivatives and thus span the upper-left and upper-right quadrants of Exhibit 2.2. Products of this type, with little or no active management, include Bridgewater Associates' "All Weather Portfolio" and Partners Group's "Diversified Beta Strategy."

Absolute return, or “pure alpha,” products are actively managed funds expected to generate returns in excess of what could be earned on simple cash but with all beta exposures hedged out (e.g., long–short market-neutral funds). The manager of an equity market-neutral fund selects equal amounts of stocks to hold long and stocks to short so that the net equity market exposure is zero. The excess return is generated by the extent to which the stocks held long outperform the market and the stocks held short underperform, independent of the broad market direction. An absolute return strategy thus falls into the lower-left quadrant of Exhibit 2.2. Alternatively, the manager might hold only individual stocks long and thus cancel out the general equity market exposure by shorting index futures contracts or swaps. This version of a market-neutral fund also ends up in the lower-left quadrant of Exhibit 2.2, but only by having positions in the upper two quadrants that cancel each other out.

Hedge funds were so named because early strategies were constructed to hedge market or beta exposure. Over time, the term has been applied to a wide variety of actively managed strategies that use a combination of leverage, shorting, and derivative positions. Many hedge funds are not pure alpha in the sense of having their beta exposures completely hedged. A single-strategy hedge fund typically has its alpha generated by active management of physical securities while the beta exposure is modified by the amount of shorting or derivatives, and thus spans the upper-right and lower-left quadrants of Exhibit 2.2.

Multiple-strategy hedge funds have been introduced in recent years. These funds use multiple strategies to generate alpha.

Funds of hedge funds combine the returns from individual hedge funds in an effort to diversify the alpha sources. These products can include positions that span all four quadrants of Exhibit 2.2.

3. Numerical Illustrations of Alpha–Beta Separation

In this chapter, we illustrate the advantages of separating alpha and beta sources of return with several numerical examples, from simple to complex. Specifically, we examine the improvements in the risk–return trade-off of a total portfolio that has the flexibility to decouple the risk exposures found in typical long-only active strategies.

Alpha–Beta Separation: Single-Fund Numerical Example

We begin with the simplest possible case: an institutional investor with only two investment vehicles—a single actively managed equity fund and cash. For purposes of illustration, we assume that the risk-free rate of return on cash is 4.0 percent and that the expected return on the actively managed equity fund is 12.0 percent. The investor believes that the expected return on the relevant index fund (e.g., the S&P 500) is 9.0 percent and is also anticipating an alpha from active management of 3.0 percent. Based on the cash return of 4.0 percent, the 9.0 percent expected return on the market index represents a 5.0 percent excess return. Of course, 9.0 percent is only the expected, or average, market return; the actual return in any given year can vary widely. We assume that the market risk of the S&P 500, as measured by the standard deviation of annual returns, is 12.0 percent. Even though the fund manager is expected to outperform the market, on average (otherwise, he or she would not have been hired in the first place), the expected alpha of 3.0 percentage points will also vary from year to year, with a standard deviation of 5.0 percent. The risk of the actively managed fund is thus higher than the 12.0 percent market risk, but not by much. Assuming that the alpha, or “active,” risk of the managed fund is uncorrelated with the market, the exact calculation is given by the Pythagorean relation $(12.0^2 + 5.0^2)^{1/2} = 13.0$ percent.⁶ We summarize the risk–return characteristics of the market, as well as the actively managed fund, by using the well-known Sharpe ratio, defined as the return in excess of the risk-free rate divided by risk. Specifically, the active fund has an expected Sharpe ratio of $(12 - 4)/13 = 0.62$, and the index fund has an expected Sharpe ratio of only $(9 - 4)/12 = 0.42$.

⁶The statistical formula for the variance (standard deviation squared) of an asset with two sources of risk is $\sigma_i^2 = \sigma_A^2 + \sigma_B^2 + 2\sigma_A\sigma_B\rho_{AB}$, similar in form to Equation A6 in Appendix A. The relatively simple calculation in this numerical example is based on the assumption that the correlation coefficient between the market and active risk components of the managed fund, ρ_{AB} , is zero and the last term of the equation drops out.

Suppose the institutional investor is a plan sponsor that requires an 8.0 percent return on funds in order to meet expected obligations. Given the risk-free rate of 4.0 percent, the investor allocates 50 percent of the total portfolio with the active equity manager described previously and 50 percent into cash; so, the expected return on the portfolio is $(0.50)12.0 + (0.50)4.0 = 8.0$ percent, as shown in **Table 3.1**. Given the 50/50 equity/cash allocation, the total portfolio has exactly half the risk of the active equity fund, or a standard deviation of 6.5 percent, as shown by the position of “50/50 Mix” in **Figure 3.1**. Can the investor do better than the 50/50 portfolio described in Table 3.1? A higher fund/cash allocation (e.g., 90/10) would increase the total portfolio expected return and risk, as shown by the position of “90/10 Mix,” but would result in the same Sharpe ratio as the 50/50 portfolio. Because one of the two assets is risk-free (i.e., cash), any mix of the managed fund and cash lies on the same reward-to-risk line shown in Figure 3.1. Leverage alone, or the lack thereof, does not change the underlying Sharpe ratio or slope of the reward-to-risk line.

Table 3.1. Basic 50 Percent Active Equity/50 Percent Cash Allocation

	Expected Return	Standard Deviation	Committed Capital Allocation	Market Exposure
Actively managed equity fund	12.0%	13.0%	50.0%	50.0%
Cash	4.0	0.0	<u>50.0</u>	<u>0.0</u>
Total portfolio	8.0%	6.5%	100.0%	50.0%
Sharpe ratio	0.62			

We now allow for the possibility of using market index futures contracts to hedge some of the risk of the actively managed fund. Specifically, we establish a short futures position to lower the market risk that the active equity fund brings to the portfolio. Note that the futures contracts are based on the market index, not on the actively managed fund. Our hedge does not impact the active risk of the equity fund but merely eliminates a portion of its market exposure.

Starting with a 90/10 fund/cash allocation, we hedge the active fund with a short index futures position that has a notional value of 60 percent of the total portfolio, as shown in **Table 3.2**. The futures position does not require any additional capital (i.e., the managed fund and cash allocations add up to 100 percent of the portfolio), although some of the cash may be required as collateral for the futures position. As explained in Appendix C, the arbitrage-based spot–futures parity condition dictates that the expected return on a *long* futures position is the 5.0 percent difference between the expected market index return of 9.0 percent and

Figure 3.1. Portfolio Risk and Return

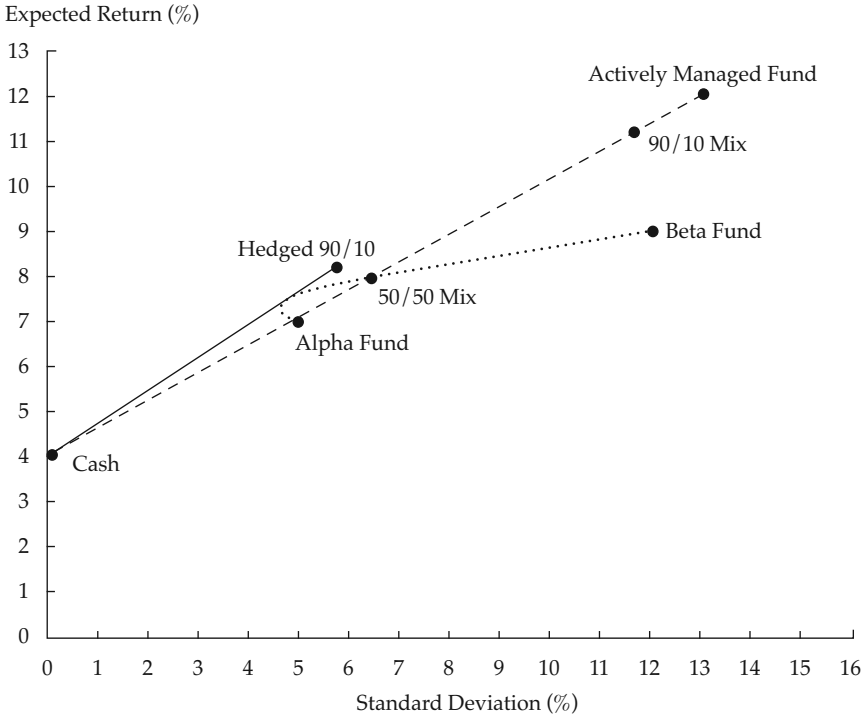


Table 3.2. Futures Hedge on 90 Percent Active Equity/10 Percent Cash Allocation

	Expected Return	Standard Deviation	Committed Capital Allocation	Market Exposure
Actively managed equity fund	12.0%	13.0%	90.0%	90.0%
Index futures	5.0	12.0	0.0	-60.0
Cash	4.0	0.0	<u>10.0</u>	<u>0.0</u>
Total portfolio	8.2%	5.8%	100.0%	30.0%
Sharpe ratio	0.73			
Hedge ratio	0.67			

the risk-free rate of 4.0 percent. Arbitrage also dictates that the risk of the futures position is the same as the risk of the market: 12.0 percent. We are interested in a *short* futures position, as expressed by the –60 percent weight, which results in a $60/90 = 0.67$ hedge ratio, as shown at the bottom of Table 3.2.

The hedged portfolio in Table 3.2 simultaneously increases the expected return and decreases the risk, as compared with the portfolio in Table 3.1, by increasing exposure to alpha and decreasing exposure to beta. The alpha is higher because the portfolio is 90 percent (instead of 60 percent) invested in the active fund. The beta is lower because of the hedge.

The improvement in Sharpe ratio, to 0.73, is shown by the higher slope of the line connecting cash to the “Hedged 90/10” portfolio in Figure 3.1. The increased reward-to-risk trade-off occurred without changing any of the fundamental assumptions about the actively managed fund or the market and without using any new source of active management (i.e., a different and better fund manager). Our somewhat arbitrary choice to hedge $60/90 = 0.67$ of the active fund’s equity market exposure yields an expected Sharpe ratio of $(8.2 - 4.0)/5.8 = 0.73$. But the maximum possible Sharpe ratio requires a slightly higher hedge ratio of 0.71, or 71 percent of the managed fund (Equation C7 in Appendix C).

We have illustrated the investor’s use of a derivatives overlay to partially separate out the alpha in the actively managed portfolio. Full alpha separation occurs when the market exposure is fully hedged (i.e., a hedge ratio of 1.0), which results in a “pure-alpha” market-neutral fund. The expected return on this market-neutral fund is the risk-free rate plus the expected alpha ($4.0 + 3.0 = 7.0$ percent), and the risk is 5.0 percent, as shown by the position of the “Alpha Fund” in Figure 3.1. To achieve the same portfolio result as in Table 3.2, the plan sponsor could also choose a 90/10 pure-alpha fund/cash allocation and a *long* index futures position of 30 percent to get the desired market exposure. A third equivalent alternative, which is instructive in terms of standard portfolio theory, is to combine the pure-alpha fund with a fully funded “beta-only” market index fund, both of which require capital, as shown in Table 3.3.

Table 3.3. Alpha (Market-Neutral) Fund and Beta (Index) Fund Portfolio

	Expected Return	Standard Deviation	Committed Capital Allocation	Market Exposure
Alpha fund	7.0%	5.0%	90.0%	0.0%
Beta fund	9.0	12.0	30.0	30.0
Cash	4.0	0.0	<u>–20.0</u>	<u>0.0</u>
Total portfolio	8.2%	5.8%	100.0%	30.0%
Sharpe ratio	0.73			

The pure-alpha and beta-only fund combination in Table 3.3 requires leverage (–20 percent cash) for enough capital to fund all the positions and to replicate the expected return and risk of the futures hedge shown in Table 3.2, but the critical point in terms of portfolio theory is the *relative* weights of the alpha and beta funds. As mentioned previously, allocations to cash (positive or negative) change the expected return and risk of the overall portfolio but do not affect its Sharpe ratio (this concept is more fully explored in Appendix A). The relative allocations to the two risky funds in Table 3.3 are $90/120 = 75$ percent to alpha and $30/120 = 25$ percent to beta. In terms of risk and return, the 75/25 alpha/beta portfolio lies on an alpha–beta “efficient frontier” curve, shown as a dotted line in Figure 3.1. According to standard portfolio theory (Equation B7 in Appendix B), the *optimal* allocation (highest Sharpe ratio) between the alpha and beta funds turns out to be 78/22. In other words, the line from cash to the “Hedged 90/10” portfolio is not quite tangent to the alpha–beta efficient frontier curve; a slightly higher slope that is perfectly tangent (not shown) crosses the efficient frontier at a 78/22 alpha/beta mix. Because it is optimal, this 78/22 alpha/beta mix would have the same maximum possible Sharpe ratio as the optimal hedge ratio of 0.71 in the derivatives–overlay strategy.

We further examine the equivalency of derivatives–overlay versus pure-alpha fund strategies by tracking the contributions of each approach to the total portfolio’s excess return and risk budget. Table 3.4 calculates the expected excess return and return variance for each component of Table 3.3 (Equations A13 and A15 in Appendix A). Note that the total portfolio excess return in Table 3.4 matches the expected excess return—8.2 percent – 4.0 percent = 4.2 percent—in Table 3.3; note also that the portfolio variance in Table 3.4 matches the squared standard deviation from Table 3.3: $(5.8 \text{ percent})^2 = 0.0033$. Table 3.4 indicates that the alpha fund contributes 64 percent of the total portfolio’s expected excess return but only 61 percent of the risk budget, which indicates that a slightly higher allocation to the alpha fund is warranted for an optimal combination. In fact, one property of an *optimal* (maximum Sharpe ratio) mix of alpha and beta sources is that the contributions of each source to the total portfolio excess return and risk are equal, as shown in Equation B8 in Appendix B. For example, at the optimal allocation between the alpha and beta (i.e., 78/22 instead of 75/25), the percentage contribution of the alpha fund to both portfolio risk and excess return is about 67 percent.

Table 3.4. Risk–Return Contributions of Combined Alpha and Beta Funds

	Excess Return Contribution	Share of Portfolio	Variance Contribution	Share of Portfolio
Alpha fund	2.7%	64%	0.0020	61%
Beta fund	1.5	36	0.0013	39
Cash	<u>0.0</u>	<u>0</u>	<u>0.0000</u>	<u>0%</u>
Total portfolio	4.2%	100%	0.0033	100%

Table 3.5 calculates the expected excess return and return variance for each component of the derivatives-overlay strategy in Table 3.2, in which the alpha and beta components of the active equity fund are listed separately. Note that the alpha component of the active equity fund in Table 3.5 has the same return and risk contributions as the pure-alpha fund in Table 3.4. But note that the pure-beta fund in Table 3.4 is equivalent to the Table 3.5 *combined* contributions of the beta components of the active equity fund and the short futures contract. Specifically, the total contribution to excess return from beta sources in Table 3.5 is $107 - 71 = 36$ percent, and the total contribution to risk from beta sources is $117 - 78 = 39$ percent. These calculations verify the equivalence of the derivatives-overlay (i.e., futures hedging) and pure alpha–beta fund (i.e., traditional portfolio theory) approaches to alpha–beta separation. As in the alpha–beta fund approach, *optimal* hedging (i.e., a hedge ratio of 0.71) has the property that the portfolio risk contribution equals the portfolio excess return contribution for each component of the strategy, as specified by Equation C7 in Appendix C.

Table 3.5. Risk–Return Contributions of Hedged Portfolio

	Excess Return Contribution	Share of Portfolio	Variance Contribution	Share of Portfolio
Actively managed equity fund (alpha)	2.7%	64%	0.0020	61%
Actively managed equity fund (beta)	4.5	107	0.0039	117
Equity index futures	-3.0	-71	-0.0026	-78
Cash	<u>0.0</u>	<u>0</u>	<u>0.0000</u>	<u>0</u>
Total portfolio	4.2%	100%	0.0033	100%

The preceding single-fund numerical example demonstrates a number of important alpha–beta separation principles (supplemented by the multifund numerical example that follows):

1. The separation of the alpha and beta components in an actively managed fund can lead to a better risk–return trade-off than can be achieved by the active fund alone, in which the alpha and beta exposures are coupled in some fixed and potentially suboptimal proportion. The separation of alpha and beta sources of return adds a degree of freedom or flexibility in portfolio structure by making possible a change in the proportion of the two components.
2. The definition of alpha as the difference in return between an actively managed fund and a market index is not simply a “relative performance” perspective on fund management. Thus defined, alpha is relevant to all market participants, given the existence of equity derivatives contracts and other forms of index exposure (e.g., index funds and ETFs) that allow for low-cost exposure replication and hedging of market returns.

3. Similarly, the importance of expected returns in excess of the risk-free rate is not an artifact of the CAPM or any other equilibrium theory of financial markets. The calculation of excess returns is important to any market participant that invests in, or borrows at, a short-term cash rate or that uses derivative securities with implied borrowing and thus an implied cash rate. Leverage, with the risk-free rate as the fulcrum point, is a principal reason that returns in excess of the risk-free rate are relevant.
4. Alpha–beta separation can be achieved by a derivatives overlay on traditional active managers or by the managers themselves in market-neutral products. The derivatives overlay requires either a *short* derivatives position added to a traditional fund or a *long* derivatives position in combination with a market-neutral alpha fund. Alpha–beta separation can also be viewed from the perspective of traditional portfolio theory (risky capital allocation) by using a combination of pure-alpha and beta-only (i.e., market index) funds. In the absence of implementation costs and fees, all three approaches—pure-alpha, beta-only, and alpha–beta funds—can be configured to produce the same proportional exposures and total portfolio Sharpe ratio.
5. The *optimal* hedge ratio in a derivatives-overlay strategy and the *optimal* allocation to separate alpha and beta funds in traditional portfolio theory produce the same result: a portfolio with the highest possible Sharpe ratio. One of the properties of optimal portfolios is the indifference between small changes in asset weights: In an optimal strategy, all assets (or asset classes) have the same marginal contribution to portfolio expected return per unit of portfolio risk.⁷ Traditional long-only strategies rarely combine the alpha and beta exposures in these optimal proportions.

Alpha–Beta Separation: Multifund Numerical Example

We now consider a numerical example of alpha–beta separation for an investor that employs several active fund managers—one each in four different asset classes. Specifically, we consider a portfolio that contains allocations to large-cap domestic equity, small-cap domestic equity, international equity, and fixed income. As previously mentioned (and explained in Equations A9–A12 in Appendix A), the additional cash allocation (positive or negative) changes the expected return and risk of a portfolio but not its Sharpe ratio. We thus focus on no-cash portfolios of risky assets (including traditional long-only actively managed funds; market index funds and/or derivatives contracts; and market-neutral, or “pure-alpha,” funds), any of which can be scaled to the desired level of total portfolio risk with an appropriate amount of cash.

⁷As indicated by Equation B4 in Appendix B, the “return” in this statement is measured in excess of the riskless rate, and “risk” is measured by variance.

Table 3.6 and **Table 3.7** contain a set of risk and return expectations for index funds in four asset categories: the S&P 500 Index, the Russell 2000 Index, the MSCI EAFE Index, and the Lehman Aggregate Bond Index. The parameter values we choose for the expected returns, standard deviations, and correlations of returns represent a set of beliefs about future market conditions informed by historical experience and other information. We continue to use 4.0 percent as the risk-free rate. The capital allocations shown in Table 3.6 are *optimal* weights based on the formulas for the mean–variance optimization of correlated risky assets (Equation B3 in Appendix B). These same optimal weights can be found by using a numerical optimizer (e.g., Excel Solver), the objective being to maximize the portfolio’s Sharpe ratio. The 0.46 Sharpe ratio for the optimal mix of passive funds (Table 3.6, shown as “Passive Portfolio” in **Figure 3.2**) is higher than the Sharpe ratio of any single index fund because the funds are not perfectly correlated: The well-known principle of portfolio diversification is at work.

Table 3.6. Optimal Portfolio of Passive (Beta-Only) Index Funds

	Expected Return	Standard Deviation	Committed Capital Allocation	Sharpe Ratio
S&P 500 Index fund	9.0%	12.0%	36.9%	0.42
Russell 2000 Index fund	11.0	20.0	11.0	0.35
EAFE Index fund	10.0	15.0	27.9	0.40
Lehman Aggregate Index fund	<u>5.0</u>	<u>6.0</u>	<u>24.3</u>	<u>0.17</u>
Total portfolio	8.5%	9.9%	100.0%	0.46

We next introduce actively managed funds in each asset class by listing their alpha characteristics in **Table 3.8**. The expected information ratios in the last column, a common measure of value added through active fund management, are calculated as the expected alpha divided by active risk (i.e., tracking error).⁸ The expected information ratios we choose are modest by most professional standards but are higher for the small-cap and international equity managers on the basis of the commonly held belief that more opportunity exists for active returns in those markets. That the expected information ratio is positive at all reflects the investor’s

⁸The information ratio, first named by Grinold (1989), is similar in form to the Sharpe ratio but is based on benchmark-relative, rather than absolute, performance. The formula for the information ratio is $IR = \alpha/TE = \alpha/\sigma_\alpha$. The information ratio equals alpha divided by tracking error, where tracking error is the standard deviation of the period-to-period alpha residuals; all variables must be expressed in consistent time units, such as annualized units. If the beta of a portfolio is zero, as with a market-neutral hedge fund benchmarked against cash, the information ratio and the Sharpe ratio are equivalent. For a more complete explanation of the information ratio, see Goodwin (1998).

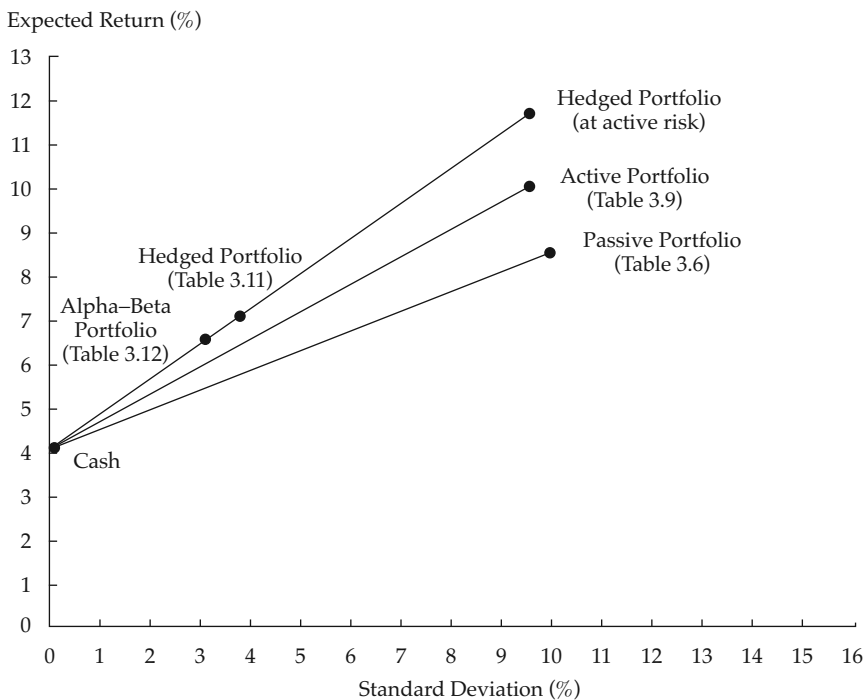
Table 3.7. Correlation Matrix of Index Funds

	SP500	R2000	EAFE	LehAgg
SP500	1.00	0.60	0.70	0.30
R2000	0.60	1.00	0.60	0.20
EAFE	0.70	0.60	1.00	0.10
LehAgg	0.30	0.20	0.10	1.00

Table 3.8. Active Fund Alphas and Active Risk

	Expected Alpha	Active Risk	Expected Information Ratio
S&P 500 fund	2.0%	8.0%	0.25
Russell 2000 fund	4.0	10.0	0.40
EAFE fund	2.0	5.0	0.40
Lehman Aggregate fund	1.0	4.0	0.25

Figure 3.2. Optimal Passive, Active, and Hedged Portfolios



belief that the manager has above-average skill (see Siegel, Waring, and Scanlan 2009).⁹ **Table 3.9** shows the total expected return (from both beta and alpha) for the actively managed funds and the total risk of each fund under the assumption that the beta and alpha risks are uncorrelated. The correlations among the actively managed funds (shown in **Table 3.10**) are also based on the assumption that the alpha risks are uncorrelated *across* funds and are thus slightly lower for each pair of managed funds than the index fund values shown in Table 3.7.

Table 3.9. Optimal Portfolio of Active Funds

	Expected Return	Standard Deviation	Committed Capital Allocation	Sharpe Ratio
Large-cap fund	11.0%	14.4%	20.6%	0.49
Small-cap fund	15.0	22.4	16.7	0.49
International fund	12.0	15.8	24.8	0.51
Bond fund	<u>6.0</u>	<u>7.2</u>	<u>37.9</u>	<u>0.28</u>
Total portfolio	10.0%	9.6%	100.0%	0.63

Table 3.10. Correlation Matrix of Active Funds

	Large Cap	Small Cap	International	Bond
Large cap	1.00	0.45	0.55	0.21
Small cap	0.45	1.00	0.51	0.15
International	0.55	0.51	1.00	0.08
Bond	0.21	0.15	0.08	1.00

The Sharpe ratio of 0.63 for the optimal mix of actively managed funds (Table 3.9) is higher than the Sharpe ratio of 0.46 for the optimal mix of index funds (Table 3.6) because of the value expected to be added by active management. This added value is shown graphically by the positions of the “Active Portfolio” and “Passive Portfolio” in Figure 3.2. But even this optimal mix of actively managed funds does not allow for the possibility of separating the alpha and beta components of each fund. We now introduce the possibility of a derivatives overlay on each of the managed funds, whereby both the hedge ratios and the fund weights are flexible (not fixed, as they are with no derivatives overlay) and are chosen to optimize the overall portfolio Sharpe ratio. Using Equations C4 and C5 in Appendix C, we

⁹*Ex post* information ratios, used in performance attribution, always vary from zero except in the unlikely circumstance that the manager had exactly the same return as the benchmark. *Ex ante*, or expected, information ratios are nonzero only if a manager is expected to outperform or underperform the market. These expectations should be developed in the context of the zero-sum nature of active management.

calculate the expected returns and risks for hedged active funds by using the optimal hedge ratios shown in the last column of **Table 3.11**. The portfolio Sharpe ratio of 0.81 in Table 3.11 is substantially higher than the value of 0.63 in Table 3.7 because hedging allows for the separation and optimal allocation of alphas and betas. The higher Sharpe ratio of the hedged portfolio is shown by the position of “Hedged Portfolio (Table 3.11)” in Figure 3.2, as well as by a hedged portfolio levered up to have the same risk as the optimal active portfolio.

Table 3.11. Optimal Portfolio of Managed Funds with Beta Hedges

	Expected Return	Standard Deviation	Committed Capital Allocation	Hedge Ratio
Large-cap fund	8.7%	10.3%	14.6%	0.460
Small-cap fund	8.9	10.3	18.7	0.875
International fund	7.0	5.5	37.4	0.841
Bond fund	<u>5.2</u>	<u>4.1</u>	<u>29.2</u>	0.822
Total portfolio	7.1%	3.8%	100.0%	
Portfolio Sharpe ratio	0.81			

To further illustrate the optimal separation of alpha and beta, **Table 3.12** shows a portfolio optimization using eight funds: four beta-only index funds and four alpha-only funds created from the traditional actively managed funds with their market exposure fully hedged away. The total portfolio Sharpe ratio of 0.81 in Table 3.12 is the same as that of the portfolio constructed with the derivatives overlay in Table 3.11 because they are both manifestations of the same principle: an improved risk–return trade-off through the separation of alpha and beta. In other words, the optimal “Alpha–Beta Portfolio” in Figure 3.2 lies on the same Sharpe ratio line as the “Hedged Portfolio (Table 3.11)” but with proportionally less expected return and risk.

Alpha–Beta Separation and Added Value

Note that the relative “beta allocations” for the index funds in the last column of Table 3.12 are the same as those in Table 3.6; the optimal portfolio of betas is preserved when the alphas are separated out of each actively managed fund. This result illustrates the “alpha–beta fund separation theorem” we describe in Appendix C (see text preceding Equation C9). Also notice the relatively large capital allocations to the alpha funds as compared with the beta funds in Table 3.12 (or equivalently, the large beta hedges in Table 3.11). When the alpha and beta sources of return in traditionally managed funds are separated, optimal portfolios often devote substantially more capital to pure-alpha sources even under the fairly modest expected information ratios listed in Table 3.8. The relatively low proportion of alpha risk in large institutional portfolios has been named the “active risk puzzle”

Table 3.12. Optimal Portfolio of Beta Funds and Alpha Funds

	Expected Return	Standard Deviation	Committed Capital Allocation	Beta Allocation	Alpha Allocation
S&P 500 Index fund	9.0%	12.0%	6.5%	36.9%	
Russell 2000 Index fund	11.0	20.0	1.9	11.0	
EAFE Index fund	10.0	15.0	4.9	27.9	
Lehman Aggregate Index fund	5.0	6.0	4.3	24.3	
S&P 500 alpha fund	6.0	8.0	12.0		14.6%
Russell 2000 alpha fund	8.0	10.0	15.4		18.7
EAFE alpha fund	6.0	5.0	30.8		37.4
Lehman Aggregate alpha fund	<u>5.0</u>	<u>4.0</u>	<u>24.1</u>		<u>29.2</u>
Total portfolio	6.5%	3.1%	100.0%	100.0%	100.0%
Portfolio Sharpe ratio	0.81				

by Litterman (2004). Litterman’s preferred explanation for this seemingly suboptimal behavior is that investors have historically been unable to separate the active (alpha) risk allocations from basic asset (beta) allocation decisions—a problem he predicts will be resolved with the use of derivative securities. Other observers of this phenomenon (e.g., Waring and Siegel 2003; Kritzman 2004) attribute the low levels of active risk in institutional portfolios to a high aversion to alpha risk versus beta risk. Plan sponsors may be less certain about the benefits of alpha sources than historical information ratios suggest given that past alpha is no guarantee of future alpha. Investors may also be more sensitive to being “wrong and alone” (alpha risk) as opposed to incurring losses that are marketwide (beta risk) and thus shared by other investors, as explained by Kritzman (1998).

The “alpha allocations” to each alpha fund in Table 3.12 are derived from Equation B7 in Appendix B, which gives optimal portfolio weights for assets, assuming the assets have uncorrelated returns (a reasonable assumption because they are pure-alpha sources) and given the active management parameter values shown in Table 3.8. For example, the allocation to the S&P 500 pure-alpha fund is proportional to its expected alpha over its active variance, $2/8^2 = 1/32$, and the allocation to the Lehman Aggregate pure-alpha fund is proportional to its expected alpha over its active variance, $1/4^2 = 1/16$. Specifically, the optimal allocation to the Lehman Aggregate alpha fund of 29.2 percent is exactly twice the 14.6 percent allocation to the S&P 500 alpha fund. Both funds have the same information ratio of 0.25, but the Lehman Aggregate alpha fund receives twice the allocation because it has half the active risk, as specified in Equation C9. The higher information ratio of 0.40 for both the Russell 2000 and the EAFE alpha funds leads to relatively higher weights for those funds as compared with the weights for the S&P 500 and Lehman Aggregate alpha funds. But the relative weights of the Russell 2000 and EAFE alpha funds are likewise 2-to-1 (37.4 percent to 18.7 percent) because the

Russell fund has twice the active risk of the EAFE fund. We note that the full set (not shown) of optimal alpha weights in Table 3.12 is unaffected by any change in beta weights, owing to modifications in the assumed market parameters. This independence of the optimal alpha and beta weight sets (based on the assumed independence of alpha and beta returns) further illustrates the alpha–beta fund separation theorem discussed in Appendix B.

Finally, we note that the Sharpe ratio of the optimal beta portfolio shown in Table 3.6 (and the top half of Table 3.12) is $(8.5 - 4.0)/9.9 = 0.46$. The optimal beta portfolio's Sharpe ratio, together with the information ratios of each of the four alpha funds, gives a total portfolio Sharpe ratio of $(0.46^2 + 0.25^2 + 0.40^2 + 0.40^2 + 0.25^2)^{1/2} = 0.81$, as specified by Equation C10 in Appendix C. Each new source of alpha increases the Sharpe ratio of the overall portfolio independent of the market or active fund from which the alpha is derived. A simpler version of this principle is also evident in the single-fund numerical example. In that example, the market had a Sharpe ratio of $5/12$ and the actively managed fund had an information ratio of $3/5$. As specified in Equation C8 in Appendix C, the highest Sharpe ratio that is possible with alpha–beta separation is a function of the market Sharpe ratio and the information ratio of the single actively managed fund: $0.71 = \sqrt{(5/12)^2 + (3/5)^2}$.

Our multifund numerical example illustrates several important principles of alpha–beta separation, in addition to the five principles already noted from the single-fund example:

6. The optimal weights of pure-alpha funds (assumed to be uncorrelated with each other and with the various beta funds) are based on their information ratios and levels of active risk. Optimal weights of *beta* exposures in a portfolio are complicated by material correlations between asset classes (e.g., domestic and international equity) but can be derived from a matrix of covariance assumptions or forecasts by using well-known portfolio optimization procedures.
7. When alpha and beta risks are uncorrelated, optimal weights of the various alpha funds are independent of the weights chosen for the beta or index funds, which are established by the overall allocation of beta risk. This “alpha–beta fund separation principle” holds whether the beta allocation is established by a formal optimizer or is based on some more subjective, *ad hoc* process for establishing the beta allocation.
8. The improved risk–return trade-off from separating alpha and beta is proportional to the square root of the sums of the squares of the information ratios of the alpha sources. Sources of alpha from any asset class or combination of asset classes (or from multiple managers in one asset class with uncorrelated alphas) add value to the overall portfolio on the basis of this mathematical relationship. The cumulative impact of several optimally weighted alpha sources can substantially increase the Sharpe ratio of the overall portfolio.

4. Calculating Alpha and Beta: Empirical Examples

The numerical examples in Chapter 3 illustrated several important principles related to the separation of alpha and beta but are simplistic in at least two ways. First, the market beta of an actively managed fund is rarely equal to exactly 1. For example, the security selection process for a large-cap domestic equity fund might be consistently biased toward stocks that are highly sensitive to marketwide movements; thus, the fund's S&P 500 beta might be 1.2. With a beta greater than 1, the simple difference between the fund return and the S&P 500 return misstates alpha because part of the apparent excess return of the portfolio over the benchmark can be replicated merely by increasing the beta—that is, through buying an S&P 500 Index fund on margin. Specifically, the market exposure of a \$100 million dollar fund with a beta of 1.2 is replicated by buying index futures contracts with a notional value of \$120 million, not \$100 million; the same market exposure is removed by shorting \$120 million of index futures contracts. Although one might argue that a high-beta fund is intentionally positioned to exploit the positive expected risk premium of the equity market, the ability to replicate the risk premium through a leveraged index fund belies the notion of true value added through active management, unless the increased beta exposure is temporary (i.e., a timing decision that is consciously part of the active management strategy). Similar misstatements of alpha occur for a fund that has a beta materially less than 1.

A second complication not covered in the hypothetical examples in Chapter 3 is that any given actively managed fund might have multiple beta exposures. For example, some actively managed equity funds have a consistent small-cap bias as compared with the S&P 500. Again, one might argue that such funds are earning alpha by exploiting the tendency for small-cap stocks to have higher returns than those of large-cap stocks. But the same permanent exposure can be obtained by an appropriate mix of S&P 500 and Russell 2000 (i.e., small-cap) index funds, without any active management. Thus, the small-cap premium earned by this passive exposure is beta, not alpha. Given the existence of equity-style indices, similar arguments can be made for funds that have a value or growth tilt. These arguments are not merely an exercise in more precise performance attribution. The point is that low-cost return replication and hedging of constant beta exposures can be used to isolate and potentially transport alpha and to ensure that active management fees are paid only for true alpha.

Equity Mutual Fund Examples

Issues concerning non-unitary, or multiple, beta exposures can be illustrated by examining the track records of several well-known mutual funds. Although the principles of alpha–beta separation apply equally as well to institutions and institutional funds, we focus on retail funds because the management philosophy and long-term return data are part of the public record. **Table 4.1** reports the historical returns for four actively managed domestic equity funds for the 10 years (120 months) from January 1998 to December 2007.¹⁰ Returns for both active funds and indices are reported in excess of the one-month Treasury bill.¹¹ For example, Table 4.1 reports that the average *excess* return for Fund A was 3.88 percent, which, together with an average annualized T-bill return of 3.49 percent, gives a *total* return of $3.88 + 3.49 = 7.37$ percent per year.

Table 4.1. Mutual Fund Annualized Returns and Risk, 1998–2007

	Fund A	Fund B	Fund C	Fund D
Average excess return over T-bills	3.88%	4.28%	3.75%	6.70%
Standard deviation	15.57	12.68	19.40	16.45
Return in excess of S&P 500	0.52	0.92	0.40	3.35

The bottom line in Table 4.1 shows that all four funds beat the return on the passive S&P 500 Index, on average, from 1998 to 2007. The characterization of these simple differences as alpha, however, is accurate only if each fund has a market beta of 1. **Table 4.2** shows the results of linear regressions of the fund returns on the S&P 500 return. In a statistical sense, the average market betas reported in Table 4.2 are only estimates of the “true” beta for each fund, which is unobservable. In particular, the full regression output (not shown) includes beta coefficient standard errors based on the sample size of 120 months. The standard errors for the beta estimates in Table 4.2 are in the range of 0.05; so, the true betas could be anywhere

¹⁰The names and tickers for the four equity funds are Fund A: Fidelity Magellan (FMAGX), Fund B: Washington Mutual Investors (AWSHX), Fund C: Janus (JANSX), and Fund D: T. Rowe Price Small-Cap Stock (OTCFX). Although the statistics we report represent actual returns on these funds over the designated periods, we do not use full names in the main text for the sake of brevity and to avoid a focus on specific commercial funds. The funds were selected because they are well-known examples of various issues involved in the calculation of alpha and beta not based on past or expected performance.

¹¹The return data are monthly observations from Bloomberg for the mutual funds and from Ibbotson Associates for the S&P 500 and T-bills. Ibbotson data are used by permission of Morningstar, Inc. The monthly means are annualized by multiplying them by 12; monthly return standard deviations are annualized by multiplying them by the square root of 12.

Table 4.2. Mutual Fund Market Betas and Annualized Alphas, 1998–2007

	Fund A	Fund B	Fund C	Fund D
Market beta	1.03	0.73	1.19	0.83
Alpha	0.41%	1.84%	−0.26%	3.91%
Active risk	3.56%	6.95%	8.39%	11.09%
Information ratio	0.12	0.27	−0.03	0.35

within ± 0.10 (two standard errors) of the reported values at the 95 percent confidence level. Thus, the beta of Fund A might arguably be 1, but the true beta of Fund B is clearly much lower than 1, and the beta of Fund C is higher than 1. We discuss beta measurement in more detail in Chapter 7, including the need for an *ex ante* estimate and the reality that fund betas change over time. Given these beta estimates, however, the realized alpha is the fund's average excess return (over the risk-free rate) minus the fund beta times the average excess return on the S&P 500. Table 4.2 also reports the realized active risk of the four funds, defined as the annualized standard deviation of the alpha returns, and the realized information ratio, defined as alpha divided by active risk.

The fact that the betas of the four funds are not all equal to 1 has an impact on the measurement of alpha. Because those impacts are sometimes small, we discuss them in terms of basis points (1 bp = 0.01 percent). For example, on the one hand, the alpha of Fund C, with its relatively high market beta, is −26 bps in Table 4.2, in contrast to the simple return difference of +40 bps in Table 4.1. On the other hand, the alpha of Fund B, with its relatively low market beta, is +184 bps, in contrast to the simple return difference of +92 bps in Table 4.1. The alpha of Fund A is fairly close to the simple return difference reported in Table 4.1 because its estimated market beta is close to 1.

As explained previously, we can separate out the alpha of each fund by hedging the market exposure through short futures contracts with a notional value based on the fund's beta. For example, the alpha delivered by a \$100 million holding of Fund B can be isolated by establishing a short position in an S&P 500 futures contract of \$73 million. The realized return on the hedged Fund B, or pure-alpha fund, over this period would have been the realized alpha of 184 bps plus the risk-free rate. Once the alpha of an actively managed fund is isolated, leverage or cash can be used to increase or decrease both the alpha and the active risk. For example, with 2-to-1 leverage, the alpha-only product based on Fund A would have an excess return of $2 \times 41 = 82$ bps, with a risk of $2 \times 356 = 712$ bps, which is in the same range as an unlevered alpha-only product based on Fund B. Given the ability (conceptual or actual) to lever or delever alpha funds, the relevant measure of added value is the ratio of alpha to active risk, or the information ratio.

We now explore the second shortcoming of the hypothetical illustrations in Chapter 3: multiple beta exposures. Although the 184 bp alpha of Fund B in Table 4.2 is impressive in contrast to the -26 bp alpha of Fund C, most mutual fund observers (e.g., Morningstar) categorize Fund B as a value-style fund and Fund C as a growth-style fund. Value funds generally pick among stocks that have low price-to-earnings ratios; growth funds pick among stocks with high earnings growth rates, which generally have high price-to-earnings ratios. Value stocks (and thus most value funds) outperformed growth stocks over the 10-year period under examination. Specifically, the Russell 1000 Value Index outperformed the Russell 1000 Growth Index at an annualized rate of 216 bps, as reported in Table 4.3. This observation might be construed as simply a statement about proper benchmarking and performance attribution, except for the fact that ETFs and other derivatives contracts are available on the separate Russell 1000-style (i.e., growth and value) indices. Thus, like the excess return on the general market, the value premium (which, over long periods of time, tends to be positive) can be hedged and replicated. Another example of a beta factor other than the general market factor for equity funds is market capitalization. Morningstar categorizes the first three mutual funds in our analysis as large cap, but Fund D is a small-cap fund. The Russell 2000 Small-Cap Index outperformed the Russell 1000 Large-Cap Index over the decade under examination at an annualized rate of 178 bps (see Table 4.3).

Table 4.3. Equity Factor Annualized Returns and Risk, 1998–2007

	Market	Small Size	Value
Average factor return	3.67%	1.78%	2.16%
Standard deviation	14.83%	12.55%	13.72%
<i>Return correlations</i>			
Market	1.00	0.04	-0.39
Small size	0.04	1.00	-0.13
Value	-0.39	-0.13	1.00

Table 4.3 provides summary statistics on three marketwide equity factors constructed from Russell index fund returns from January 1998 to December 2007. “Market” is the annualized return on the Russell 1000 Index in excess of the risk-free rate, where the general term market refers specifically to the large-capitalization domestic equity market. “Small Size” is the annualized return on the Russell 2000 Index minus the return on the Russell 1000 Index. “Value” is the return on the Russell 1000 Value Index minus the return on the Russell 1000 Growth Index. Although one could conduct a regression analysis of the various Russell size and style indices directly, we focus on return *differences*, implemented through long and short index

derivatives, to separate the size and value exposures from the general market exposure. For example, on the one hand, the Small-Size factor is generally unrelated to the Market factor, as indicated by the low correlation coefficient of 0.04 in Table 4.3. On the other hand, the Value factor has a material negative correlation of -0.39 with the Market factor (e.g., value stocks tend to underperform growth stocks when the general market is up), even though our definition of the Value factor (the difference between two large-cap domestic equity indices) might suggest that the factor is uncorrelated with the general market. In an ideal world, the various beta factors would be independent (i.e., correlation coefficients close to zero), but the beta factors used in practice generally have material nonzero correlations.

Table 4.4 reports on multifactor regressions of the four funds in Table 4.2 (using the equity factors described in Table 4.3). For example, Fund A's market beta of 0.99 remains close to 1, similar to the single-factor market beta reported in Table 4.2.¹² Fund A has no material Small-Size exposure (estimated value of -0.02) and only a slightly negative Value exposure (estimated value of -0.10). Because the market beta remains close to 1 and the additional factor betas are close to zero, the Fund A alpha of 48 bps in Table 4.4 is little changed from the single-factor alpha estimate of 41 bps in Table 4.2.

Table 4.4. Mutual Fund Multifactor Betas and Annualized Alphas, 1998–2007

	Fund A	Fund B	Fund C	Fund D
Market	0.99	0.89	1.05	0.86
Small size	-0.02	-0.09	0.12	0.76
Value	-0.10	0.50	-0.39	0.08
Alpha	0.48%	0.07%	0.53%	2.00%
Active risk	3.20%	2.80%	5.66%	4.44%
Information ratio	0.15	0.02	0.09	0.45

The multifactor story for Fund B is more interesting. Fund B has a large Value beta of 0.50 (standard error of 0.02), and the market beta has increased as compared with the single-factor analysis in Table 4.2. The combined effect of these two estimates is a substantial reduction in alpha: only 7 bps in Table 4.4 as compared with 184 bps in Table 4.2. We again emphasize that the various beta and alpha estimates are not merely an exercise in more precise performance attribution, although this kind of regression analysis is a helpful tool for measuring the added value in actively managed portfolios. In particular, Fund B's return of almost 1 percentage point per

¹²The switch of the general market factor from the S&P 500 to the Russell 1000 between Tables 4.2 and 4.4 is inconsequential. Specifically, the correlation between these two large-cap domestic equity indices was 1.00 (calculated to two significant digits) from 1998 to 2007.

year higher than the S&P 500 for 10 years is impressive on its face, but the multifactor regression analysis in Table 4.4 indicates that most of this extra return (i.e., all but 7 bps) could have been obtained by appropriate beta exposures to various Russell index funds. Fund C, however, has a large *negative* Value exposure and is thus appropriately categorized by Morningstar as a growth-style fund. Because the returns for Fund C were earned during a period when growth stocks generally underperformed, the alpha after hedging is fairly high. This result is partially offset by the fact that Fund C had a slight small-cap bias (size beta of 0.12) during a period when small-cap stocks outperformed, but the annualized alpha of 53 bps in Table 4.4 is still large and positive, in contrast to the -26 bp alpha in Table 4.2.

Our fourth mutual fund, Fund D, is actually a small-cap fund and would typically not be benchmarked against large-cap indices like the S&P 500 or the Russell 1000. We intentionally included a small-cap fund in our example to illustrate the power of regression analysis in identifying multiple beta factors. Although Fund D's alpha of 391 bps (Table 4.2) is extraordinary, it is suspect in a single-factor regression against the Russell 1000 Large-Cap Index during a period when small-cap stocks outperformed. The multifactor regression in Table 4.4 reveals a significant small-size exposure (size beta of 0.76) and a substantial reduction in the measured alpha (to 200 bps). Once the small-cap nature of Fund D is properly identified, the estimate of active risk is also reduced, from 11.09 percent (Table 4.2) to 4.44 percent (Table 4.4). This result illustrates a general principle: As meaningful beta factors are added to an analysis of fund returns, both the absolute magnitude of alphas and the level of active risk tend to be reduced.¹³

Are the general market, small-size, and style factors the only relevant betas in equity fund returns? What factors *should* be included in a regression analysis of active fund returns? The inclusion of size and value factors, in addition to the general equity market, is now fairly common in the analysis of domestic equity funds. This practice has been formalized in the mutual fund industry by the Morningstar classification system and canonized in financial economics by the Fama–French three-factor model (Fama and French 1993). More to the point, highly liquid ETFs and derivatives contracts based on size and style indices facilitate low-cost hedging and return replication for these factors. But what if a fund also has exposures to other asset classes (e.g., international equity or fixed income)? The return patterns in international equity and fixed income are just as multifaceted as those in domestic equity and are unlikely to be captured by merely adding one or two factors. Furthermore, what are the relevant factors for such alternative assets as private

¹³In linear regression analysis, even meaningless factors added to the right-hand side of the regression equation will reduce the variance of the residuals on which the active risk number is calculated. Leaving out important market factors (e.g., size for a small-cap fund) leads to artificially high estimates of active risk. Econometric methods are available to test whether the reduction in residual variance from adding independent variables in a linear regression is material.

equity, real estate, and commodities? Fung and Hsieh (2001) are among the many authors who have studied this question. They used an extensive list of possible beta factors to analyze the returns associated with hedge funds, as shown in **Exhibit 4.1**, and then added option and trend following factors for each category.

Exhibit 4.1. Fung and Hsieh Hedge Fund Factors

Equity	S&P 500, FTSE 100, DAX 30, Nikkei 225, Australian All Ordinary
Fixed income	U.S. 30-year, U.K. gilt, German Bund, French 10-year, Australian 10-year
Interest rates	Eurodollar, 3m Sterling, Euro-DM, Euro-Yen, Australian Bankers Acceptance, Paris Interbank Offered Rate
Currency	British pound, German mark (now replaced by the Euro), Japanese yen, Swiss franc
Commodities	Corn, wheat, soybeans, crude oil, gold, silver, Goldman Sachs Commodity Index, Commodity Research Bureau Index

Returning to funds that are primarily invested in domestic stocks, we find that the momentum factor provides an interesting case study. The momentum effect, first documented in academic finance journals by Jegadeesh and Titman (1993), is that stocks that have performed well over the last several months tend to continue to outperform in the current month. Carhart (1997) helped formalize the common definition of momentum used in practice: the return of a stock over the last year, excluding the most recent month (i.e., an 11-month return). Portfolios of stocks ranked by momentum show impressive performance characteristics over the long term, and momentum is an important element in the strategy of many successful money managers (see Mulvey and Kim 2008). Indeed, momentum has now been canonized in the academic literature as a potential fourth factor by receiving its own Fama-French acronym, UMD (for “up minus down” stocks).

Should momentum be considered a beta exposure and added to the list of standard equity factors? The answer is both critical and controversial from a performance attribution perspective. Historical statistics on the performance of the momentum effect are quite strong; the return premium in backtested momentum portfolios is at least as high as the better-known small-size and value premiums and generally more consistent. Active portfolio managers that overweight momentum stocks, however, might argue that their explicit or implicit awareness of the momentum effect is part of their managerial added value. At some point, the trend toward finding every characteristic of stocks that has paid off in the past and designating them *ex post* as beta factors makes recognizing any positive alpha impossible. From a financial economics perspective, the momentum factor is suspect given the lack of a clearly defined and nondiversifiable risk factor, as required for a positive risk premium in equilibrium models. Although financial economists

acknowledge the impressive historical performance statistics of momentum, they generally attribute the phenomenon to behaviorally induced market inefficiencies or regard it as simply an exercise in data mining. From the replication and hedging perspective, which is the primary motivation of our examination of alpha and beta, the momentum factor does not have well-established indices and thus does not have ETFs or derivative securities. We thus leave as an open question the designation of momentum as a beta factor.

Fixed-Income Fund Examples

For a second empirical illustration of the separation of alpha and beta, we turn to the fixed-income market and actively managed bond portfolios. **Table 4.5** reports the annualized average excess returns and standard deviations of three bond funds for the 10 years (120 months) from January 1998 to December 2007.¹⁴ Table 4.5 also reports the simple difference between the returns on the actively managed funds and the Lehman Aggregate Bond Index, similar to the comparison of the active equity funds and the S&P 500 returns in Table 4.1. For example, Fund E's excess return of 2.39 percent happens to match exactly the excess return on the Lehman Aggregate Index, and thus the return difference on the last line of Table 4.5 is zero.

Table 4.5. Bond Fund Annualized Returns and Risk, 1998–2007

	Fund E	Fund F	Fund G
Average excess return	2.39%	2.90%	3.44%
Standard deviation	3.56%	5.09%	5.12%
Return, Lehman Aggregate	0.00%	0.51%	1.05%

As observed previously for the equity market, simple differences between the portfolio return and the benchmark return are valid approximations of alpha only if the managed funds have a fixed-income beta of 1. The single-factor regressions in **Table 4.6** show that a fixed-income beta of 1 is a reasonable estimate for Fund E but not for Funds F and G. Because Fund F's estimated beta (0.57) is low, it has a relatively high alpha of 154 bps, as shown in Table 4.6, well above its simple return difference of 51 bps in Table 4.5. But Fund G, with its relatively high estimated beta of 1.12, has an alpha that is lower than the simple difference in Table 4.5. As is typical of fixed-income funds, the alphas and active risks are lower than for the actively managed equity funds in Table 4.2, but the sizes of the information ratios are similar.

¹⁴The names and ticker symbols for the three bond funds are Fund E: JPMorgan Core Bond (WOBDX), Fund F: Managers Bond (MGFIX), and Fund G: Oppenheimer Strategic Income (OPSIX). As with the equity funds, we do not use these names in the main text for the sake of brevity and to focus on principles as opposed to specific fund track records.

Table 4.6. Bond Fund Market Betas and Alphas, 1998–2007

	Fund E	Fund F	Fund G
Beta	1.01	0.57	1.12
Alpha	−0.03%	1.54%	0.78%
Active risk	0.76%	4.73%	3.43%
Information ratio	−0.04	0.32	0.23

As in the equity market, the correct list of fixed-income factors is a matter of opinion. The Morningstar classification system for bond funds and bond analysts recognizes two fixed-income factors: duration and credit. Price sensitivity to changes in interest rates, as measured by duration, is the dominant source of risk for fixed-income securities. We define the Duration factor as the return on the Lehman Treasury (all maturities) Index in excess of the risk-free rate as measured by one-month T-bills. As reported in Table 4.7, the Duration factor returns are almost identical to the excess returns on the Lehman Aggregate Bond Index, with a correlation coefficient of 0.96. Thus, when we put in the underlying factors for fixed income (Duration, Credit, and Prepayment), the Lehman Aggregate “factor,” an aggregation of the three major underlying factors plus some others, drops out.

Table 4.7. Fixed-Income Factor Annualized Returns and Risk, 1998–2007

	Lehman Aggregate	Duration	Credit	Prepayment
Average factor return	2.39%	2.37%	0.04%	0.70%
Standard deviation	3.44%	4.47%	2.58%	2.15%
<i>Return correlations</i>				
Lehman Aggregate	1.00	0.96	−0.07	0.35
Duration	0.96	1.00	−0.29	0.19
Credit	−0.07	−0.29	1.00	0.54
Prepayment	0.35	0.19	0.54	1.00

The second most important source of uncertainty in fixed-income securities is credit or default risk. We measure returns to the Credit factor as the difference between the monthly return on the Lehman Corporate Investment Grade Index and the Lehman Treasury Index. Although similar, the Duration factors of these two fixed-income indices are not an exact match, which leaves open the possibility that the Credit factor also includes some duration risk. The payoff to the Credit factor, as we define it, of 4 bps over the period under study (reported in Table 4.7) is quite small compared with its long-term historical and future expected payoffs.

We also include a Prepayment (bond call option) factor. Specifically, bonds and mortgages often include a call feature that allows the issuer to repurchase the bond or prepay the mortgage at a prespecified price should interest rates drop. We measure the Prepayment factor as the difference between the Lehman Investment Grade CMBS and the Lehman U.S. Treasury three-to-seven-year index returns. The CMBS (commercial mortgage-backed security) index captures the risk of early mortgage payoff, an option for owners who wish to refinance.¹⁵ We subtract the returns on the Treasury index (with a three-to-seven-year maturity) because it reasonably matches the duration of the CMBS index, although we recognize that the CMBS duration can change over time. This attempt to control for duration leads to a relatively small correlation of 0.19 between the Prepayment and Duration factors, as reported in Table 4.7. Unfortunately, the CMBS index still has material credit risk, which results in a fairly high correlation of 0.54 between the Prepayment and Credit factors. As observed previously for the equity market, beta factors in an ideal world would be completely independent to facilitate a clean interpretation of estimated exposures. This outcome, however, is not achievable for fixed-income factors, given the available data.

Table 4.8 reports the regressions of the three actively managed bond funds on the Duration, Credit, and Prepayment fixed-income factors. For comparison purposes, we also include a regression of Lehman Aggregate index returns on the various fixed-income factors in the last column of Table 4.8. Fund E has a large Duration beta of 0.78, a small Credit beta of 0.17, and a statistically insignificant Prepayment beta of 0.08 (the standard error on this beta estimate is 0.05). Consistent with Fund E's general fixed-income beta of close to 1, as reported in Table 4.6, the beta exposures for Fund E are quite similar to the Lehman Aggregate index.

Table 4.8. Bond Fund Multifactor Betas and Alphas, 1998–2007

	Fund E	Fund F	Fund G	Lehman Aggregate
Duration	0.78	0.51	0.89	0.78
Credit	0.17	1.47	1.02	0.26
Prepayment	0.08	-0.01	0.27	0.09
Alpha	0.47%	1.64%	1.09%	0.47%
Active risk	0.94%	3.44%	2.63%	0.51%
Information ratio	0.50	0.48	0.42	0.93

¹⁵We look to the mortgage market for returns on the Prepayment factor because mortgage borrowers almost always have the right to prepay without a penalty, unlike in the corporate debt market, where the borrower may or may not have the right to prepay.

Funds F and G are even more interesting fixed-income examples. Fund F has a substantially lower Duration exposure than does Fund E, which explains Fund F's low beta with respect to the general fixed-income market reported in Table 4.6. Fund F also has a large Credit factor exposure of 1.47, which indicates that the fund invests in lower-rated bonds than does Fund E. In a period when the Credit factor payoff was high, this Credit beta would have resulted in a significant reduction in estimated alpha, but the impact was generally insignificant during the 1998–2007 period because the Credit factor payoff turned out to be only 4 bps, on average. Fund F appears to have little exposure to the Prepayment factor. Fund G has a slightly higher duration than the market as a whole, with a Duration factor estimate of 0.89 as compared with 0.78 for the Lehman Aggregate index; and it has a very high Credit factor beta, similar to that for Fund F. Unlike the other two funds, Fund G also has a statistically significant Prepayment factor beta of 0.27.

Interestingly, the Lehman Aggregate index itself has small but nonzero alpha and active risk numbers in Table 4.8, which suggests that fixed-income factors other than Duration, Credit, and Prepayment may be at play or that the indices we have chosen only approximately replicate the beta exposures. Given that the Lehman Aggregate index shows an alpha of 47 bps and that one can invest in a Lehman Aggregate index fund, the measured alphas of the three active bond funds are arguably 47 bps too high. Finally, we note that the information ratios for the three active bond funds in our example are fairly similar to the equity funds. For example, Fund G's information ratio of 0.42 indicates that, with appropriate leverage, the fund's potential for adding to overall portfolio alpha matches that of the small-cap fund (Fund D) in Table 4.4.

Hedge Fund Strategy Examples

For a third empirical example of the separation of alpha and beta, we look at hedge fund returns, with the caveat that we examine hedge fund indices (groups of funds) rather than individual funds. Table 4.9 reports on four hedge fund indices supplied by Hedge Fund Research, Inc. (HFRI) for the same 10-year period, 1998–2007, for which we examined equity and fixed-income funds. The excess return statistics for all four hedge fund indices are impressive. For example, the broadest hedge fund index, the Fund Weighted Composite Index, had an annualized excess return of

Table 4.9. Hedge Fund Index Annualized Returns and Risk, 1998–2007

	Fund Weighted Composite	Fund of Funds Conservative	Quantitative Directional	Distressed Restructuring
Average excess return	6.28%	2.93%	8.33%	6.98%
Standard deviation	7.16%	3.41%	14.22%	5.80%

6.28 percent (total return of 9.77 percent with the risk-free rate included) and a standard deviation of 7.16 percent—about half the risk of the general equity market. In addition to high returns, a common argument for hedge fund strategies is that most of the capital market risks, or betas, have been “hedged”; so, the returns are primarily driven by alpha sources. The hedge fund regressions that we examine next indicate that this assertion is only partially true for hedge funds in the aggregate.

In **Table 4.10**, we regress the excess return of the various hedge fund indices in Table 4.9 on five previously used factors: three Russell-based equity factors and two Lehman-based fixed-income factors. The Market beta estimate of 0.32 for the Fund Weighted Composite Index indicates that some of the aggregate hedge fund returns are attributable to, and could be replicated by, general equity market exposure. The Small-Size factor beta of 0.24 is also material, which indicates that some of the aggregate hedge fund returns benefited from an exposure to the small-cap premium, which can also be replicated. The Fund Weighted Composite has a slightly negative Value factor exposure (hedge funds in the aggregate favored growth stocks), although this exposure was not material enough to affect measured alpha. The Fund Weighted Composite does not have significant exposure to the fixed-income factors, Duration and Credit; although the 0.19 coefficient on Credit might appear large, it is not statistically significant. Note that the standard deviations of the Duration and Credit factor returns in Table 4.7 are small in relation to those of the equity (Market, Small-Size, and Value) factor returns in Table 4.3. Because the various factors have different standard deviations, the regression coefficient magnitudes among factors are not directly comparable. Over time, factors with smaller return standard deviations will have both larger estimated coefficients and larger coefficient standard errors.

Table 4.10. Hedge Fund Multifactor Betas and Annualized Alphas, 1998–2007

	Fund Weighted Composite	Fund of Funds Conservative	Quantitative Directional	Distressed Restructuring
Betas				
Market	0.32	0.08	0.71	0.14
Small size	0.24	0.07	0.43	0.15
Value	-0.06	-0.02	-0.16	0.04
Duration	0.04	-0.01	0.08	0.02
Credit	0.19	0.28	0.35	0.67
Alpha	4.69%	2.60%	5.09%	6.03%
Active risk	3.42%	2.79%	4.42%	4.42%
Information ratio	1.73	0.93	1.15	1.37

The second HFRI index in Tables 4.9 and 4.10 is an aggregation of funds of hedge funds rather than the hedge funds themselves. The hedge fund ideal of little to no replicable factor exposures is more closely met by the Fund of Funds Conservative Index, which has low betas on the equity factors, although the Credit factor coefficient of 0.28 is statistically significant (standard error of 0.12). The lower betas indicate that only a small portion of the 2.93 percent excess return is lost to beta factor exposures, which leads to an alpha estimate of 2.60 percent. The “conservative” aspect of this index results in fairly low active risk as compared with the composite hedge funds. The Fund of Funds Conservative Index has an information ratio of 0.93, which is lower than the 1.73 value for the Fund Weighted Composite, perhaps owing to the added level of fees in funds of funds.

The second and third examples of hedge fund indices in Tables 4.9 and 4.10 track two specific hedge fund categories: Quantitative Directional and Distressed Restructuring. HFRI defines Quantitative Directional as funds that use sophisticated quantitative models to select long and short positions in equity securities but that do not hedge out all equity market risk (thus, the term “directional”). The high estimated Market factor beta of 0.71 in Table 4.10 indicates that these funds tend to be net-long, and the relatively high Small-Size and negative Value factor betas indicate that the Quantitative Directional funds favor small-cap and growth-style stocks. Indeed, the equity factor betas of the aggregate hedge fund index in the first column of Table 4.10 may be specifically a result of the presence of Quantitative Directional and other equity-based hedge fund strategies in the aggregate. HFRI defines Distressed Restructuring as an event-driven strategy that focuses on the fixed-income and equity securities of corporations near or in bankruptcy and reorganization. As a result, the Distressed Restructuring Index in Table 4.10 shows, as one would expect, a large and highly significant beta of 0.67 on the Credit factor, as well as small but statistically significant betas on several other factors.

Although the reported alphas on all four hedge fund indices in Table 4.10 are lower than the raw excess returns in Table 4.9, the beta factors are low enough that a large portion of the hedge fund returns appears to be unrelated to replicable domestic capital market factors and survives as measured alpha. The relatively low active risk levels lead to impressive information ratios for the hedge fund strategies as compared with the information ratios for the equity and fixed-income funds in prior tables. We note, however, that the hedge fund indices may be subject to selection biases that overstate the alpha of the average hedge fund, and beta factors outside the U.S. capital markets are used in some hedge fund categories (e.g., emerging markets).¹⁶

¹⁶Specifically, hedge funds with poor track records sometimes do not report their returns to a database provider.

The regression analysis of the hedge fund indices naturally leads to the question of how much each strategy is devoted to alpha versus beta sources of return. If the hedge funds are not all alpha, as suggested above, are they at least mostly alpha? One good way to measure the relative contributions of alpha and beta sources of return and risk is an *ex post* version of the *ex ante* risk-budgeting and contribution analysis described in Appendix A (see Equations A13–A15.) For example, Table 4.11 uses return data from Table 4.1 and the multifactor regression results from Table 4.4 to calculate the return and risk contributions of the four domestic equity funds. The return contribution from alpha is simply the alpha divided by total excess return, with the rest of the realized excess return being attributed to beta exposures. The risk contribution from alpha is active risk squared (i.e., active variance) divided by the total risk squared, with the rest of the realized fund risk being attributed to beta exposures. In a full *ex post* risk-budgeting analysis, the risk contribution from the collective beta factors could be further decomposed into the risk contribution from each factor. Also note that in this *ex post* regression analysis, the total realized variance of each fund is exactly the sum of the active return variances and the beta return variances because regression alphas are perfectly uncorrelated with estimated betas.

Table 4.11. Alpha and Beta Contributions of Mutual Funds, 1998–2007

	Fund A	Fund B	Fund C	Fund D
Total excess return	3.88%	4.28%	3.75%	6.70%
Total risk	15.57	12.68	19.40	16.45
Alpha	0.48	0.07	0.53	2.00
Active risk	3.20	2.80	5.66	4.44
<i>Return contribution</i>				
Beta factors	88%	98%	86%	70%
Alpha	12	2	14	30
<i>Risk contribution</i>				
Beta factors	96%	95%	91%	93%
Alpha	4	5	9	7

Table 4.11 confirms the conventional wisdom that traditionally managed equity mutual funds are mostly beta, with less than 10 percent of total risk coming from alpha exposure for each of the four funds. The contribution to total excess return by the alpha exposures is also low, although it is higher than 10 percent in two cases and as high as 30 percent for Fund D. In contrast to the equity mutual funds, the hedge fund index returns decomposed in Table 4.12 (based on data from Tables 4.9 and 4.10) show that a large majority of the total excess return comes

Table 4.12. Alpha and Beta Contributions of Hedge Funds, 1998–2007

	Fund Weighted Composite	Fund of Funds Conservative	Quantitative Directional	Distressed Restructuring
Total excess return	6.28%	2.93%	8.33%	6.98%
Total risk	7.16	3.41	14.22	5.80
Alpha	4.69	2.60	5.09	6.03
Active risk	3.42	2.79	4.42	4.42
<i>Return contribution</i>				
Beta factors	25%	11%	39%	14%
Alpha	75	89	61	86
<i>Risk contribution</i>				
Beta factors	77%	33%	90%	42%
Alpha	23	67	10	58

from alpha exposure—as high as 89 percent for the Fund of Funds Conservative Index. The alpha contributions to total hedge fund risk are also high—as high as 67 percent for the Fund of Funds Conservative Index—but not as high as the return contribution in each index (i.e., 67 percent compared with 89 percent).

Empirical Review of Beta Factors

The choice of beta factors for the domestic equity and fixed-income markets has been greatly influenced over the years by the academic research of Eugene Fama and Kenneth French (1993). Although the Fama–French factors are not defined with respect to tradable market indices, they do provide a long-term perspective on the return characteristics of U.S. financial markets. **Table 4.13** reports return statistics, including Sharpe ratios, on six Fama–French factors over the decade examined throughout this chapter (1998–2007). Although primarily motivated by academic research, the Fama–French equity factors in Table 4.13 are quite similar to the tradable factors based on Russell indices that we define in Table 4.3. Specifically, over the decade under study, the correlation between MKT and “Market” is 0.99, the correlation between SMB and “Small Size” is 0.92, and the correlation between HML and “Value” is 0.89. Currently, no tradable indices exist for UMD, or “Momentum,” stocks, although indices and associated derivatives may be introduced over time on the basis of market demand. Likewise, the two Fama–French fixed-income factors are similar to tradable factors based on Lehman indices. Specifically, the correlation between TERM and “Duration” is 0.96, and the correlation between DEF and “Credit” is 0.80. The Fama–French fixed-income factors do not include a version of the Prepayment factor.

Table 4.13. Fama–French Factor Annualized Returns and Risk, 1998–2007

	MKT	SMB	HML	UMD	TERM	DEF
Average factor return	4.30%	3.29%	4.43%	10.34%	3.94%	-0.65%
Standard deviation	15.46%	14.58%	13.38%	19.87%	8.91%	3.76%
Sharpe ratio	0.28	0.23	0.33	0.52	0.44	-0.17
<i>Return correlation</i>						
MKT	1.00	0.26	-0.52	-0.24	-0.27	0.51
SMB	0.26	1.00	-0.50	0.22	-0.07	0.20
HML	-0.52	-0.50	1.00	-0.07	0.11	-0.14
UMD	-0.24	0.22	-0.07	1.00	0.14	-0.31
TERM	-0.27	-0.07	0.11	0.14	1.00	-0.54
DEF	0.51	0.20	-0.14	-0.31	-0.54	1.00

Notes: MKT is excess equity market—the return on the broad U.S. equity market (valued-weighted composite of all stocks in the CRSP database) minus the one-month T-bill return. (CRSP is the Center for Research in Security Prices at the University of Chicago, which maintains a comprehensive long-term database on all U.S. stocks for purposes of academic research.) SMB is “small minus big” or small-size premium—the return on a CRSP portfolio consisting of all the stocks in the CRSP database and constructed to be long small-cap stocks and short large-cap stocks. HML is “high minus low” market-to-book ratio, or value premium—the return on a CRSP portfolio that is long value stocks and short growth stocks. UMD is “up minus down” price, or momentum premium—the return on a CRSP portfolio that is long stocks with high past returns and short stocks with low past returns. TERM is the term-structure factor for the U.S. fixed-income market—the return on long-term Treasury bonds minus the one-month T-bill return (from Ibbotson Associates). DEF is the default factor for the U.S. fixed-income market—the return on long-term corporate bonds minus the return on long-term Treasury bonds (from Ibbotson Associates).

Even over a 10-year period, the reported factor returns may not be indicative of the long-term history or expectations. **Table 4.14** reports on the Fama–French factors in **Table 4.13** but does so over the 50-year period that precedes the 1998–2007 decade. Notable differences include the long-term historical value for MKT (the general U.S. equity market return in excess of the risk-free rate) of 8.12 percent, which leads to a very high Sharpe ratio of $8.12/14.27 = 0.57$. Many market observers believe that this historical value, which captures the period of very strong growth in the U.S. economy and stock market from 1948 to 1998, is unsustainable. The 10.13 percent return on UMD in **Table 4.14**, the long-term momentum factor, is even more extraordinary, with a Sharpe ratio of $10.13/10.48 = 0.97$.

Although we have focused on empirical examples of domestic equity and fixed income in this chapter, the asset management world extends far beyond the U.S. capital markets. The equity and fixed-income markets in other countries are as complex and multidimensional as the U.S. market, particularly in light of currency risk considerations. In addition, such alternative assets as real estate, commodities, and private equity, with subcategories in each asset class, are potential beta factors, even though some may not have fully satisfactory indices or tradable derivatives. Although we do not examine actively managed funds in these other markets,

Table 4.14. Long-Term Fama–French Factor Annualized Returns and Risk, 1948–1997

	MKT	SMB	HML	UMD	TERM	DEF
Average factor return	8.12%	1.43%	4.77%	10.13%	1.05%	0.27%
Standard deviation	14.27%	8.81%	8.42%	10.48%	8.75%	3.87%
Sharpe ratio	0.57	0.16	0.57	0.97	0.12	0.07
<i>Return correlation</i>						
MKT	1.00	0.26	−0.23	0.03	0.28	0.05
SMB	0.26	1.00	−0.09	−0.16	−0.10	0.11
HML	−0.23	−0.09	1.00	−0.15	−0.02	0.08
UMD	0.03	−0.16	−0.15	1.00	0.02	−0.13
TERM	0.28	−0.10	−0.02	0.02	1.00	−0.46
DEF	0.05	0.11	0.08	−0.13	−0.46	1.00

Table 4.15 provides summary statistics for various asset classes over the 1998–2007 decade.¹⁷ Table 4.15 includes only broad factors: GSCI (commodities), NCREIF (real estate), CAPE (private equity), and FX (currencies) can each be broken down into various subcategories, as in the U.S. equity and fixed-income markets. In Table 4.15, the Sharpe ratio of 1.00 for the private equity index is impressive, and the Sharpe ratio of 4.39 for real estate is extraordinary. Of course, the returns to these two asset classes were unusual in the most recent decade, which helps explain why they have become so popular among investors. Note that because the various equity indices are not defined in excess of a global equity common factor, the returns are highly correlated. The correlation with the S&P 500 is 0.88 for the Russell 2000, 0.91 for the EAFE, and 0.73 for the private equity index. The EAFE-weighted basket of foreign currency exposures, FX, was positive, on average, over the decade, but in contrast to the other factors, it may not have a positive expected excess return over the long run.

Although the broad index returns in Table 4.15 do not all have the same long-term history as the Fama–French factors in Table 4.14, we do report on the prior decade in **Table 4.16**. Notable differences as compared with the most recent decade in Table 4.15 include the negative 10-year excess return on real estate (positive total return but less than the risk-free rate) and the positive correlation between the fixed-income and equity markets. Indeed, the large negative correlation between equity and fixed-income returns reported in Table 4.15 is found mostly in the most recent decade; it is anomalous in a long-term perspective except for a brief period in the late 1950s.

¹⁷The returns on the real estate and private equity indices are available only on a quarterly basis; thus, the statistics in Table 4.15 for all asset classes are based on quarterly returns, which may vary slightly from statistics based on monthly returns in previous tables. In addition, reported returns on real estate and private equity indices are subject to smoothing as compared with the highly liquid capital market returns, which affects the accuracy of the standard deviation and correlation estimates.

Table 4.15. Broad Factor Annualized Returns and Risk, 1998–2007

	S&P500	R2000	EAFEH	LAFI	GSCI	NCREIF	CAPE	FX
Average	3.66%	5.81%	5.38%	2.40%	4.42%	8.85%	11.15%	1.52%
Std. dev.	16.70%	21.60%	18.18%	3.46%	23.31%	2.02%	11.17%	8.04%
Sharpe ratio	0.22	0.27	0.30	0.70	0.19	4.39	1.00	0.19
<i>Correlation</i>								
S&P500	1.00	0.88	0.91	-0.52	0.10	0.25	0.73	-0.03
R2000	0.88	1.00	0.87	-0.49	0.31	0.11	0.69	-0.05
EAFEH	0.91	0.87	1.00	-0.61	0.18	0.33	0.77	-0.20
LAFI	-0.52	-0.49	-0.61	1.00	-0.01	-0.19	-0.50	0.27
GSCI	0.10	0.31	0.18	-0.01	1.00	0.03	0.16	-0.24
NCREIF	0.25	0.11	0.33	-0.19	0.03	1.00	0.53	0.03
CAPE	0.73	0.69	0.77	-0.50	0.16	0.53	1.00	-0.02
FX	-0.03	-0.05	-0.20	0.27	-0.24	0.03	-0.02	1.00

Notes: S&P500 is excess return on the S&P 500 Large-Cap Domestic Equity Index. R2000 is excess return on the Russell 2000 Small-Cap Domestic Equity Index. EAFEH is excess return on the currency-hedged MSCI EAFE International Equity Index. LAFI is excess return on the Lehman Aggregate U.S. Domestic Fixed-Income Index. GSCI is excess return on the S&P/Goldman Sachs Commodity Index. NCREIF is excess return on the NCREIF Real Estate Index. (NCREIF is the National Council of Real Estate Investment Fiduciaries.) CAPE is excess return on the Cambridge Associates Private Equity Index. FX is return on the EAFE minus the EAFEH return.

Table 4.16. Prior-Decade Broad Factor Annualized Returns and Risk, 1988–1997

	S&P500	R2000	EAFEH	LAFI	GSCI	NCREIF	CAPE	FX
Average	12.15%	9.60%	3.50%	3.65%	7.83%	-0.60%	10.70%	-1.11%
Std. dev.	10.43%	17.11%	15.94%	4.53%	13.24%	3.37%	5.81%	9.60%
Sharpe ratio	1.16	0.56	0.22	0.81	0.59	-0.18	1.84	-0.12
<i>Correlation</i>								
S&P500	1.00	0.79	0.68	0.49	0.19	0.02	0.34	-0.18
R2000	0.79	1.00	0.69	0.27	0.20	-0.03	0.33	-0.39
EAFEH	0.68	0.69	1.00	0.16	0.13	0.16	0.30	-0.29
LAFI	0.49	0.27	0.16	1.00	-0.11	-0.23	0.16	0.23
GSCI	0.19	0.20	0.13	-0.11	1.00	-0.01	0.08	-0.21
NCREIF	0.02	-0.03	0.16	-0.23	-0.01	1.00	0.27	-0.20
CAPE	0.34	0.33	0.30	0.16	0.08	0.27	1.00	-0.24
FX	-0.18	-0.39	-0.29	0.23	-0.21	-0.20	-0.24	1.00

The empirical analysis in this chapter has revealed a number of important concepts about the separation of alpha and beta, in addition to the basic principles illustrated by the numerical examples in Chapter 3. These additional concepts can be summarized as follows:

1. Beta exposures in actively managed funds are not always equal to 1. Both hedging and replication of the beta component of returns in a fund require an estimate of the fund's beta. A fund with a beta greater or less than 1 requires hedging a dollar exposure that is correspondingly greater or less than the net asset value of the fund.
2. Many actively managed funds have multiple beta exposures, not merely a single beta, with respect to the general market. For example, equity funds may include style and size tilts that can be hedged or replicated by using combinations of various style and size indices. Ideally, multiple beta factors would be independent of each other, although in practice, beta returns generally have material positive or negative correlations with each other.
3. Although the correct list of beta factors in any given asset class is a matter of choice, it is largely driven by the perspective (e.g., financial economics, performance attribution) from which alpha and beta are viewed. Our focus on hedging and replication—critical to the physical separation of alpha and beta sources of return—requires that a beta factor have exchange-traded indices and associated derivative securities in order to be considered. If such indices and derivatives are present, one can implement alpha–beta separation strategies.
4. Beta sources of return extend beyond the U.S. equity and fixed-income markets to international capital markets, currencies, commodities, real estate, and any other asset categories that have a tradable contract. Each of these major asset categories has subcategories and geographic regions, which lead to a wide array of possible beta factors in the global market.

5. Portable Alpha Applications

In this chapter, we review examples of how various institutions, primarily pension plan sponsors, have incorporated alpha and beta concepts into their investment management practices. Basic information is summarized in **Table 5.1**, followed by commentary on each institution's innovative or otherwise interesting practices in alpha–beta separation.¹⁸ Table 5.1 shows that the institutions vary substantially in size (assets under management), the advent of alpha–beta separation practices, sources of alpha, and implementation of beta exposure. Two of the institutions are affiliated with U.S. states (Massachusetts and Vermont), one is from Canada (Ontario), and two are from Europe (the Netherlands and Sweden).

Ontario Teachers' Pension Plan

The Ontario Teachers' Pension Plan (OTPP) formally began operations in 1990 but has existed in previous forms since 1917. One of the largest institutional investors in Canada, OTPP has more than \$100 billion in assets and administers the pensions of about 250,000 active and retired teachers in the province of Ontario. Bob Bertram, executive vice president of investments at OTPP, described the evolution of OTPP's portfolio management philosophy as follows:

When we started in 1990, the fund had about \$15 billion in assets, primarily invested in nonmarketable, nontransferable government bonds. We examined our liabilities, and they looked like 20-year duration bonds, 100 percent indexed to inflation, creating an asset/liability mismatch. Real-rate bonds like TIPS in the U.S. and RRBs in Canada were just coming into play and had little volume, so we decided to go into equity, initially through index funds.¹⁹

OTPP conducted an extensive strategic review starting in 1995 and concluded, among other things, that index funds alone were unlikely to meet the 5.25 percent real return target needed to meet our liabilities, so we moved to active equity management to add value. This sequence of events probably provided us a clearer understanding than other plans that active fund returns are independent of index returns. We then started looking at how we could supply the index portion of the

¹⁸We use plan sponsors to provide real-world illustrations of alpha–beta separation principles to avoid the commercial bias potentially associated with product providers (i.e., funds and investment management consultants). Although the examples in this monograph are provided with the plan sponsors' permission, the focus on the alpha–beta separation practices of these institutions and any remaining factual errors are the authors' alone.

¹⁹TIPS are U.S. Treasury Inflation-Protected Securities. RRBs are Canadian Real Return Bonds.

Table 5.1. Plan Sponsor Summaries

	OTPP	TKP	PRIM	AP3	VPIC
Fund size (US\$ billions)	\$108.5	\$15.7	\$53.2	\$34.8	\$3.2
Portable alpha inception	1995	1999	2006	2005	2005
Portable alpha (% of total fund)	33%	8%	5%	All liquid assets	13%
Alpha sources	Various sources, including traditional long-only managers	Geographically diverse long-short equity managers and GTAA	Hedge funds among several strategy categories	Pure-alpha equity long-short, GTAA, and traditional long-only	Portable alpha from fixed income and equity volatility
Beta instruments	Swaps	Futures	Swaps and futures	Swaps and futures	Swaps and futures
Beta from swaps	100%	0%	85%	50%	18%
Beta from futures	0%	100%	15%	50%	82%
Internal beta management	Yes	Yes	No	Both	No

Notes: OTPP is the Ontario Teachers' Pension Plan. (OTPP uses a risk-budgeting process that separates the active management risk from the passive or beta risk of its assets. The 33 percent number quoted in the table under OTPP represents the portion of total portfolio risk resulting from active management.) TKP is TKP Investments BV. PRIM is Massachusetts Pension Reserves Investment Management. AP3 is the buffer fund for the Swedish National Pension. VPIC is the Vermont Pension Investment Committee.

program ourselves. We found we could get our S&P 500 exposure for just a few basis points through a future or swap that required little capital. We decided to add value by buying hedge funds to stack on top of the index. Our process was to do a long–short swap out of the bonds into cash to buy the hedge funds and use the hedge funds as collateral for the S&P 500 Index exposure. That’s the genesis of our portable alpha program.

Bertram added that OTPP’s early adoption of what are now widely accepted alpha–beta separation concepts facilitated a transition into the risk-budgeting process that guides the plan today:

We go to our board and ask them to approve a passive risk level: how much beta risk we can have in the fund. They approve that number and also the amount of risk we want to allocate to our alpha generation strategy. If we want to hire a traditional manager who doesn’t conform to our risk-budgeting process, we go ahead and hire them on a standard contract and break the risk down internally. Our first priority for outside managers is that they have skill, independent of their embedded beta. When we hire an outside manager, we are in the position to simply strip the beta out of the portfolio ourselves. Our internal groups are then measured against their risk budgets and return targets, with respect to the overall portfolio.

TKP Investments BV (the Netherlands)

TKP Investments BV is a business unit of AEGON, one of the largest exchange-listed insurance companies in the world. TKP’s core business is the provision of investment management services to about two dozen Dutch pension funds. TKP acts as the lead investment manager and advises on asset allocation and other aspects of investment policy for its client pension plans. TKP is organized into several departments devoted to manager selection (equities, fixed income, and alternatives), real estate, and fiduciary management.

Coos Luning, head of the Multi-Manager Investments Department at TKP, explained that TKP first began to implement portable alpha strategies in 1999. TKP chose portable alpha as an efficient means of achieving return targets and meeting pension liabilities. Initially, TKP purchased beta from outside managers, but it now creates beta internally with futures contracts. TKP takes a global geographic approach with regard to alpha generation, as Luning explained:

We allocate portions of the alpha portfolio to different regions of the world. For example, we have an external long–short equity manager that covers Europe and another that covers Japan. When these regional portfolios are added up, we have a global alpha portfolio with less of a chance of overlap in investments. The benefit from separating mandates on a regional basis is the relatively uncorrelated sources of alpha return.

As demonstrated in Chapter 3, uncorrelated sources of alpha return are a key driver for obtaining a high reward-to-risk ratio in the overall portfolio.

As in the Ontario Teachers' Pension Plan, the separation of alpha and beta facilitates TKP's overall portfolio risk-budgeting process. Luning explained why the distinction between alpha and beta sources of risk is important:

We have a risk budget for alpha returns and a risk budget for beta returns. We keep them separated because they are quite different in nature. Beta risk is rewarded to anyone willing to take it, while alpha risk is only rewarded when you select the right manager.

Massachusetts Pension Reserves Investment Management

Pension Reserves Investment Management (PRIM) is responsible for the supervision of the Pension Reserves Investment Trust (PRIT) fund. PRIT is a pooled fund of the Massachusetts State Teachers' and Employees' Retirement Systems, as well as smaller municipal retirement systems in Massachusetts that choose to invest in the fund. PRIT was created by the Massachusetts legislature in December 1983 with a mandate to grow assets through investment earnings in order to reduce the state's unfunded pension liability and to assist local participating retirement systems in meeting their future pension obligations.

PRIM funded its portable alpha program in September 2006, with three fund-of-hedge-funds managers (Crestline Investors, Grosvenor, and Strategic Investment Group) and one beta overlay manager (Russell Investments), and allocated 5 percent of PRIM's \$53 billion in total assets. Russell Investments uses a hybrid approach of about 85 percent swaps and 15 percent futures for its S&P 500 beta overlay. The fund-of-hedge-funds managers are benchmarked against LIBOR plus 3 percent, and the beta manager is benchmarked against the S&P 500 minus LIBOR; thus, the combined program is evaluated against the S&P 500 plus 3 percent. The domestic equity beta source and benchmark, however, were changed from the S&P 500 Index to the broader Russell 3000 Index in May 2008. The three fund-of-hedge-funds managers currently have investments in more than 100 underlying hedge funds, with aggregate strategy allocations of 34 percent market neutral, 30 percent equity long-short, 23 percent event driven, 4 percent macro, and 9 percent other strategies.

Stan Mavromates, PRIM's chief investment officer, explained the motive behind PRIM's portable alpha strategy as follows:

The markets in 2001 and 2002 made us go back to the drawing board to see if there was anything that we could do to reduce the volatility of the fund assets and returns. As part of that review, we cut domestic equities from 42 percent down into the 20s and introduced an absolute return allocation. The reallocation was motivated by the lack of consistent alpha from long-only managers, especially in domestic equity.

Hannah Comross, PRIM's senior investment officer, commented on the subsequent evolution of PRIM's strategy:

Initially, we spent a lot of time educating the board that hedge funds and portable alpha strategies reduce instead of increase the risk of the overall portfolio. More recently, we've been happy to remind them about the risk reduction in the current asset allocation, compared to our portfolio in 2001–02.

Apparently, PRIM's board has been successfully convinced of the virtues of alpha–beta separation. In August 2008, the board announced the termination of several long-only active managers as part of a major strategic shift to index (pure-beta) funds and portable alpha funds.

Buffer Fund for the Swedish National Pension

AP3 is one of five so-called buffer funds in the Swedish national pension system, with about 210 billion Swedish kroner (US\$34.8 billion) in assets as of June 2008. AP3's capital, together with that of three other buffer funds, is used to balance deficits that temporarily arise between pension contributions and pension disbursements in Sweden. AP3 carried out a review of its portfolio structure in 2006, with the goal of making decisions on market exposure (beta) independent from active management (alpha). The stated goals in making the distinction between alpha and beta were to (1) make the portfolio structure more flexible, (2) make both the alpha and beta portfolios work harder, (3) improve diversification, and (4) reduce costs in order to increase after-cost returns. Eric Valtonen, AP3's chief investment officer, explained that AP3 was part of the emergence of alpha and beta concepts throughout the European pension system:

Alpha–beta separation in Europe became popular by 2004. Things were in the air and we exchanged ideas at conferences, including brainstorming discussions with the other pension funds. By early 2005, we had developed a good understanding of the concept at AP3. At first, it was focused on packaging hedge funds with a beta wrapper. Then, the concept broadened and people started thinking in terms of alpha and beta in the other asset classes. The first step was what we called alpha–beta awareness of return streams. From this perspective, the traditional long-only paradigm may not be the optimal way of doing things. First, you are limited to quite narrow alpha sources—basically, active stock picking. Second, cheaper and more flexible alternatives to managing beta become available.

AP3 is an interesting case study in that both its organizational structure and its portfolio policy are based on the separation of alpha and beta. In June 2008, AP3 announced a change in its organizational structure, from one based on asset classes to departments based on managing alpha and beta. Valtonen described the new organizational structure as follows:

The culmination in June of this year was when we reorganized our people. We don't have equity or fixed-income teams anymore. We now have an alpha team and a beta team. If you have the proper people working on beta and alpha separately, you can do a better job at creating an optimal beta portfolio and an optimal alpha portfolio. For example, part of our beta team is a treasury department that takes care of all of the cash transactions, funding transactions, synthetic exposures, etc. Obviously, any large pension fund will have some cash management duties, but three years ago, we explicitly realized that we would need a proper treasury group to centralize these management activities of the fund.

Vermont Pension Investment Committee

The Vermont Pension Investment Committee (VPIC) oversees the assets of the State of Vermont's public pension plans, including the Vermont State Teachers Retirement System, the Vermont State Retirement System, and the Vermont Municipal Employees Retirement System, as well as one smaller fund. Each fund has a slightly different asset allocation that is based on its funded status. The State Teachers fund, for example, is the most aggressive and has a higher equity allocation than the others because it is currently underfunded. The combined value of the defined-benefit plans is \$3.2 billion as of 30 June 2008.

VPIC has adopted an alpha-beta separation perspective through a specific portable alpha mandate. The first request for proposal and manager search was conducted in 2004. The formal mandate stated:

Portable alpha products must have minimum assets under management of \$200 million, be bundled complete with the alpha source and beta management, and possess a live, GIPS-compliant, alpha track record that is at least three years long. The use of hedge fund strategies for any part of the portable alpha mandate will not be considered.

As of June 2008, VPIC had hired two portable alpha managers (PIMCO and Oppenheimer Capital) and is currently working on a third. David Minot, PIMCO's director of finance and investment for the office of the treasurer, provided a review of the external manager strategies from a client perspective:

PIMCO captures beta through S&P 500 Index futures with a margin requirement of about 5 percent or 6 percent. For alpha, they use their well-known fixed-income capabilities, which are pretty complex, including everything from short-term fixed income, interest rate swaps, and currency swaps to credit default swaps. Oppenheimer Capital gets its fixed-income beta through a LIBOR-based swap; they pay

LIBOR on our behalf and get the Lehman Aggregate index return. For alpha, they essentially sell a put on the equity market (S&P 500 or other indices) and partially offset the volatility risk by buying a cheaper put at a lower strike. Over the long run, the theory is that folks who buy equity put contracts want market protection and are really risk averse, so they pay high premiums for downside protections.

VPIC hires external portable alpha managers because of its relatively small size. As Minot noted, “We, by necessity, have to look for a turnkey product where they provide both beta and alpha sides of the investment.” Multiple external managers help ensure that the alpha sources are relatively independent. VPIC places its portable alpha managers in traditional asset classes on the basis of the beta exposure rather than in a separate “portable alpha” category. The quest for alpha sources is independent of the beta exposure. For example, VPIC, like most plans, has a large U.S. equity allocation but little expectation of alpha from large-cap U.S. stocks. As Minot observed:

We’ve looked at portable alpha for our domestic [U.S.] large-cap allocation but haven’t considered it yet for other asset classes like small-cap domestic, international, or global fixed income. In our view, you can get alpha returns in these less efficient markets through traditional means: good research and security selection. On the other hand, we believe the large-cap equity market is pretty efficient. Similarly, you are not going to easily beat the core fixed-income benchmark of the Lehman Aggregate through traditional alpha sources. So, those are the categories where you have to think outside the box to achieve alpha.

Applications Summary

The perspectives of these five plan sponsors are representative of a number of other institutions we contacted while writing this monograph. Almost everyone we spoke with was aware of the basic concept of alpha–beta separation, and many had either implemented or were considering specific mandates for beta, pure–alpha, or portable alpha strategies. An appreciation for the distinction between alpha and beta sources of return in portfolio management was pervasive among the professional investment staff and officers, although each institution had a slightly different take on the most important conceptual takeaways and practical implications. The general perspective was that the alpha–beta dichotomy has emerged as a major theme in asset management practice over the last decade, similar to such other conceptual frameworks as liability-driven investing (LDI) and portfolio risk budgeting. Some viewed the emergence of alpha–beta separation perspectives as comparable in importance to modern portfolio theory or an appreciation of the role of asset correlations and diversification in earlier decades.

The most commonly stated challenge in implementing alpha–beta separation was board education. Board members without investment management backgrounds found the inherent jargon and quantitative principles somewhat inaccessible. Much

of the current research being published in professional journals assumes a basic awareness of the alpha–beta dichotomy and addresses advanced debates on what labels should be assigned to products and strategies that lie somewhere in the middle of the alpha–beta spectrum. Some board members were wary of strategies that used derivative securities, even if the derivatives were used to hedge risk or obtain standard market exposures in a more cost- and capital-efficient way. Given the recent concerns about counterparty and settlement risks in derivatives markets, the wariness is justified but can be mostly overcome by diversification among counterparties and by the use of counterparties on whom the investor has performed due diligence.

6. Implementation Issues

Our primary focus in this monograph is the conceptual understanding of alpha–beta separation principles, including historical context, terminology, numerical illustrations, and current examples. The implementation of portable and pure-alpha strategies requires expertise in derivative securities management and quantitative techniques that are beyond this monograph’s scope. Ample coverage of the derivative securities market, with applications to hedging and replication, is available from other sources, including materials published by CFA Institute. Similarly, numerous textbooks, as well as CFA Institute publications, provide in-depth coverage of the quantitative techniques used in portfolio management. Our goal in this chapter is to introduce some of the issues associated with the implementation of alpha–beta separation in institutional portfolios. We briefly cover the search for alpha sources, management of derivatives-based beta exposures, measurement issues associated with alpha and beta, and liquidity considerations.

The Search for Alpha

Alpha is a scarce commodity in the financial markets. As explained by Sharpe (1991), active management is a zero-sum game in which alpha is earned by one investor at the expense of others. Although the zero-sum game perspective is intimidating enough, financial economists also assert various forms of the efficient market hypothesis, whereby realized alphas are mostly random—that is, a result of luck rather than skill. The informational efficiency of securities markets continues to be a subject of debate, but active managers and those who evaluate them are the first to admit that track records, no matter how impressive, provide little guarantee of future alpha.

To illustrate this point by using inferential statistics, we consider the EAFE-benchmarked manager in Table 3.8, who has an expected alpha of 2 percent per year and a tracking error of 5 percent. Those values constitute an impressive information ratio of $2/5 = 0.4$, which is based on the twin beliefs that (1) the market in question is inefficient, and (2) the manager in question has above-average talent. A statistician hoping to verify the manager’s skill at producing alpha in this market would start with the “null hypothesis” of an average alpha of zero. The statistician needs to observe an active return track record with a t -statistic of about 2.0 or higher to verify historical above-average talent at the standard 95 percent confidence level. The formula for the t -statistic in this portfolio management application is

$$t\text{-Statistic} = \frac{\alpha - \alpha_0}{TE / \sqrt{T}}, \quad (6.1)$$

where α is the 2 percent alpha of our hypothetical EAFE manager, TE is the 5 percent tracking error, α_0 is the 0 percent null hypothesis, and T is the number of years of track record.²⁰ Solving for the value of T for a t -statistic of 2.0 suggests that the statistician needs 25 years of observable track record to be confident that the manager is above average. Observers rarely have the luxury of waiting a quarter century before deciding to hire a fund manager. Clearly, a track record alone cannot be the sole criterion for finding alpha.

The search for alpha requires a subjective evaluation of management personnel and portfolio strategy, in addition to objective data on past performance. Market expertise and experience are helpful but may not be sufficient to justify a positive expected alpha in a zero-sum game against other professional investors. Just as fundamental equity analysts look for firms with a strategic competitive advantage in their industry, plan sponsors look for managers and strategies that have an identifiable competitive advantage in asset management. The manager search process in institutional settings can be quite lengthy, involving consultants, requests for proposal, site visits, and other due diligence procedures beyond the scope of this monograph, but at least three ideas can be specifically derived from alpha-beta separation principles.

First, many academic market observers admit that the degree of informational efficiency varies with the structure of the market. For example, no one in the ivory tower would claim that the market for used cars is perfectly efficient in the sense that posted prices represent the best estimate of fair value. Markets with decentralized trading, unregulated information, lack of shorting and arbitrage, and low transaction volume are arguably less efficient and thus have better prospects for added value through active management. One of the key concepts in alpha-beta separation is that the quest for alpha does not need to be limited to the asset categories and allocations specified in the overall portfolio policy. For example, plan sponsors might maintain a significant allocation to large-cap U.S. equity without believing they have access to positive-alpha managers in that market. Alpha from a less competitive market, such as emerging market equities, can be transported and attached to the large-cap U.S. equity asset class or simply left as a separate “pure-alpha” component of the overall portfolio. As explained in Chapter 3, alpha sources with the highest information ratios provide the best added value, independent of the asset category from which they are derived.

²⁰This simple statistical exercise assumes that the realized, or “sample,” values of the alpha and tracking error parameters happen to be equal to their “population” values. Extensions of this exercise indicate that the key parameter is the information ratio, in that results do not depend on the amount of leverage. In addition, it can be shown that the results are independent of the measurement frequency. For example, quarterly rather than annual observations of the manager’s performance still require a quarter century of track record for statistical confidence.

Second, the focus on alpha and active risk in portfolio management theory has led to an increased awareness of the impact of portfolio constraints, as explained in Clarke, De Silva, and Thorley (2002). Portfolio strategies that remove constraints that bind other participants provide one possible source of competitive advantage. If most of the competitors in a particular market are long-only, then long-short strategies provide more flexibility in structuring portfolios to exploit mispricing. Other constraints (e.g., socially responsible investing [SRI] or sector neutrality and security weight limits used as risk management tools) may also limit the competitive advantage of some active managers; other active managers can take advantage of the opportunities or inefficiencies produced by these constraints. The extent to which a given constraint provides other managers with opportunity must be determined on a case-by-case basis, but the focus on alpha as a separable component of portfolio return highlights the strategic implications of portfolio constraints.

Third, the search for alpha requires a careful assessment of managerial fees. Positive expected alpha is of little value to the plan sponsor if it does not exceed the fees charged to provide it. A major academic study by French (2008) recently set the average cost of active management in the U.S. equity market at 67 bps of assets under management (AUM); these costs are not recovered by active investors *in the aggregate*. In other words, active management is a zero-sum game among all market participants before costs but is a slightly negative-sum game net of costs. Macroeconomic theory suggests that active securities management is an important social good in that accurate pricing causes scarce capital to be allocated to its best and highest societal use. But these economic gains accrue to society as a whole, not to the average active investor.

Although a general analysis of management fees and trading costs is outside the scope of this monograph, the separation of alpha and beta can provide important perspectives on costs. As noted by Kritzman (2007) and others, fees for pure-alpha sources of return cannot be directly compared with fees for traditional portfolio management, which includes a mixture of alpha and beta. Consider, for example, the numerical example described at the beginning of Chapter 3. The total expected return on the traditional actively managed fund is 12 percent, which is based on a 4 percent risk-free rate, a 5 percent market risk premium, and a 3 percent expected alpha. The risk of the actively managed fund is 13 percent, which is based on a 12 percent market risk and a 5 percent uncorrelated alpha risk. Suppose that this hypothetical manager hedges the market component of the fund and offers a “high-octane” hedge fund to clients by using 12-to-1 leverage. The expected return on the hedge fund is the 4 percent risk-free rate + $12 \times 3 = 40$ percent, and its risk is $12 \times 5 = 60$ percent. These are admittedly high risk and return numbers that are presented to make a point, but they are consistent with the manager’s underlying information ratio of $3/5 = 0.6$ —that is, 36 percent *excess* return divided by 60 percent risk.

Typical hedge fund fees are a “flat,” or *ad valorem*, fee of 200 bps of AUM plus a 20 percent performance fee (i.e., one-fifth of the realized positive return). An assessment of performance fees requires complicated stochastic analysis; thus, for our illustration, we replace performance fees with a much higher assumed AUM fee of 500 bps.²¹ To keep the arithmetic simple, we also assume that the fees are paid with a separate check, not out of the fund return. Suppose that the fee for the traditionally managed fund is the previously quoted U.S. equity average of 67 bps and that the fee for the relevant market index fund is 10 bps. Is the hedge fund fee of 500 bps outrageous by comparison? Consider a portfolio that is composed of a 90 percent index (beta-only) fund and a 10 percent hedge (alpha-only) fund. The expected return on this combination is $(0.9)(9) + (0.1)(40) = 12.1$ percent, slightly higher than that of the traditionally managed fund; the risk is $[(0.9^2)(12^2) + (0.1^2)(60^2)]^{1/2} = 12.4$ percent, which is slightly lower and thus roughly comparable to that of the traditionally managed fund (though slightly better). The weighted average fee on this 90/10 hedge/index fund combination is $(0.9)(10) + (0.1)(500) = 59$ bps, a modest bargain over the 67 bps charged for the traditionally managed fund with similar risk and return.

The point of this simple numerical example is that the seemingly small fees associated with traditionally managed funds can be quite high per unit of alpha (not beta) produced. This insight comes from the understanding that most of the return in a traditional active fund comes from the market (beta) component. Alternatively, seemingly high hedge fund fees can be a bargain if the funds are a high-information-ratio source of “true” alpha: alpha that is positive in realization as well as expectation and uncorrelated with the market return. Because expected alpha and active risk scale with leverage, fees based on assets under management may be inappropriate. A more economically justified way to compare fees is to calculate the ratio of the dollar fee paid to the dollar amount of realized alpha, a framework more or less consistent with the current “performance fee” structure of the hedge fund industry.²² To make the reward for alpha returns commensurate with the risk taken, Coleman and Siegel (1999) have further suggested charging a fee proportional to the realized information ratio (alpha divided by active risk).

²¹A hedge fund with the fee schedule described above would charge a total of 500 bps if it had a one-year return of 15 percent because $0.02 + (0.2 \times 0.15) = 0.05$; thus, a 500 bp fee assumption for a 40 percent one-year return is very conservative.

²²Nevertheless, there are at least two theoretical problems with the performance fee structure of the hedge fund industry. First, fees are generally asymmetric—that is, they are charged only for positive realized returns without a rebate for negative realized returns, although a “high-water mark” provision can provide a partial or, in some cases, full rebate. Second, to avoid charging a fee on the riskless part of the return, fees should be assessed on returns in excess of the risk-free rate rather than on total returns. With a risk-free rate of 4 percent, a hedge fund manager with a 20 percent performance fee based on total returns can invest in cash and still earn 80 bps of AUM as a performance fee, plus any applicable flat fee.

Beta Management

One of the common myths associated with alpha–beta separation is that beta exposures are cost and hassle free. Although market or beta exposure is cheap in comparison with alpha, beta returns are not entirely free of costs or management issues. In this section, we briefly examine some of the costs and management challenges associated with beta sources of return in institutional portfolios. Specifically, we discuss the relative costs of various forms of beta exposure, as well as the tracking error associated with derivatives-based beta.

Market beta exposure can be obtained through a variety of different instruments, the specifics of which depend on the particular market under consideration. Perhaps the best-known beta return is large-cap U.S. equity as measured by the S&P 500. **Table 6.1** provides a review of the costs quoted by a major investment bank for a \$1.5 billion one-year exposure to the S&P 500 in April 2008. The alternatives for S&P 500 beta include physical exposure through purchasing the 500 individual stocks (i.e., constructing an index fund) and various kinds of derivatives exposure, including exchange-traded funds (ETFs), futures, and swaps. Table 6.1 includes two examples of ETFs that track the S&P 500: the SPDR (“Spider”) ETF (ticker symbol SPY) and the iShares S&P 500 ETF (ticker symbol IVV). Table 6.1 also includes two S&P 500 futures contracts traded on the Chicago Mercantile Exchange: the standard contract, with a notional value of \$250 times the index, and the newer e-Mini contract, with a notional value of \$50 times the index. The last column of Table 6.1 quotes the costs of a customized swap contract for a \$1.5 billion one-year exposure to S&P 500 Index returns.

The round-trip (entry and exit) transaction costs for S&P 500 beta exposure in Table 6.1 include relatively small brokerage commissions (first row). The second row adds the larger “market impact” costs, which are based on estimated movements in bid and ask prices associated with assimilating a \$1.5 billion transaction. For example, the highest round-trip transaction cost estimate is 75.3 bps for the standard S&P 500 futures contract and the lowest is 31.4 bps for the e-Mini futures contract. Although the e-Mini contract’s notional value is lower, it has higher volume because trading is electronic (as opposed to open outcry), which allows for lower transaction costs.

Table 6.1 also shows that each alternative for beta exposure, including direct physical exposure through owning a basket of S&P 500 stocks, has some form of holding cost. Although only 2.2 bps, direct S&P 500 Index exposure through holding individual stocks requires rebalancing associated with changes in the composition of the index and dividend payments over a one-year period. The annual management fees of 9.4 and 9.0 bps for the ETFs are also relatively low for the S&P 500 Index, in contrast to ETFs that track other equity indices. Interestingly, the “holding cost” for both futures contracts is negative on this particular date, which possibly reflects a difference between the interest rate embedded in the futures price

Table 6.1. Transaction and Holding Costs for Six Alternatives for S&P 500 Exposure
(in basis points)

	Stocks	ETF		Futures		Swap
		SPDR	iShares	Regular	e-Mini	
Commission	14.6	4.3	4.3	0.3	1.4	10.0
Market impact	<u>30.8</u>	<u>30.8</u>	<u>30.8</u>	<u>75.0</u>	<u>30.0</u>	<u>30.8</u>
Transaction costs	45.4	35.1	35.1	75.3	31.4	40.8
Securities rebalancing	2.2					
ETF management fee		9.4	9.0			
Futures basis				-13.7	-9.6	
Swap rate spread						5.0
Holding cost	2.2	9.4	9.0	-13.7	-9.6	5.0
Total long	47.6	44.5	44.1	61.6	21.8	45.8
Incremental short	<u>30.0</u>	<u>16.2</u>	<u>25.6</u>	<u>30.0</u>	<u>30.0</u>	<u>8.0</u>
Total short	77.6	60.7	69.7	91.6	51.8	53.8

Note: Estimates are based on \$1.5 billion one-year exposure in April 2008.

and current market interest rates. Finally, the swap contract also has a holding cost, which is based on a 5 bp differential between the interest rate embedded in the contract and LIBOR. We will return to the impact of alternative interest rate quotes and interest rate movement when we discuss beta tracking error.

The sum of the transaction and holding costs in Table 6.1 represents the total cost of beta exposure under each alternative for one year, although the transaction costs would be amortized if a constant beta exposure were held for several years. For example, even with its negative holding cost, the regular futures contract has the highest *total* S&P 500 exposure cost of the six alternatives (61.6 bps).

The bulk of Table 6.1 is associated with *long* beta exposure (e.g., in market replication), but the hedging required in many portable alpha strategies requires a *short* beta exposure. The next-to-last row in Table 6.1 shows estimates for the incremental (excess over long) cost of a \$1.5 billion *short* exposure under each alternative. For example, mispricing of the futures contract works against short positions on this particular date, and short positions in the ETFs entail a “haircut” cost, as with any other stock. Indeed, avoiding the incremental costs of shorting is one of the motivations for the reunion of alpha and beta in long–short extension strategies, as discussed in Chapter 7. The key takeaway from Table 6.1 is that beta returns may be cheap in comparison with alpha but are not entirely cost free.

We now turn to the issue of tracking-error risk inherent in beta exposures obtained through futures contracts, sometimes called “synthetic” exposures. In theory, a long position in an equity futures contract, together with an investment in cash at the risk-free rate, provides the same return as a direct investment in the stocks that compose the index. The equivalence of the synthetic or futures-based return and the actual return on any index basket is based on arbitrage as expressed in the spot–futures parity condition. Specifically, Equation C1 in Appendix C states that the futures price is equal to the current spot price adjusted by the risk-free rate, net of dividends. As explained in Appendix C, one result of this parity condition is that the theoretical return on the futures contract is the return on the market index minus the risk-free rate. Thus, the return on a long futures plus cash combination should equal the total return on the market index.

Several real-world reasons explain why actual beta returns on futures contracts do not exactly equal beta returns obtained by using physical securities. First, a futures contract that exactly matches the desired market exposure may not exist. For example, in the U.S. equity market, futures contracts are available on both the S&P 500 and the Russell 2000. In contrast, an investor attempting to gain exposure to the Japanese equity market as represented by the MSCI Japan Index does not have an equivalent futures contract. The available futures contracts in the Japanese equity markets are tied to the Tokyo Stock Price Index and the Nikkei 225, both of which have different compositions than the MSCI Japan. An investor seeking to replicate or hedge an MSCI Japan beta is forced to use a “cross-hedge” on one of the available futures contracts, which can result in annual tracking errors in excess of 2 percent.

Second, even if the futures contract matches the desired index, the expiration date of the futures contract rarely coincides with the desired length of the beta exposure. Futures contracts are typically rolled over several times to obtain even a one-year exposure, and the ending date of the desired exposure generally requires getting out of the final contract early. These rollover and timing mismatches lead to various forms of “basis risk” or tracking error between the spot index and the futures price. Ultimately, accumulated basis risks are the result of the fact that interest rates change over time, the dividend estimates used in pricing the contract are imprecise forecasts, the interest rate yield curve is not flat, and transaction costs prohibit perfect arbitrage between the spot and futures markets. Indeed, even in the absence of significant transaction costs, arbitrageurs with limited capital, risk aversion, and various execution constraints may not always arbitrage futures and spot prices to exact parity. The spot–futures parity condition for futures contracts that enables synthetic beta exposures to be reasonably precise is not dictated by regulatory constraint or governmental mandate; it only holds given the participation of well-capitalized arbitrageurs in a close-to-frictionless market. **Table 6.2** provides a list of actual tracking errors for synthetic exposure to a number of equity market indices, as reported by a major investment bank in July 2008. The reported tracking errors (differences between synthetic and physical returns) are based on monthly observations over the prior 36 months.

Table 6.2. Equity Index Synthetic Exposure Tracking Errors
(annualized standard deviation of differences to physical return, in percentage points)

Domestic U.S. Index	Tracking Error	International Non-U.S. Index	Tracking Error
S&P Midcap	0.23	FTSE 100	0.84
Russell 2000	0.55	Nikkei 225	1.14
DJIA	0.86	Hang Seng	1.67

A specific example of tracking error associated with synthetic S&P 500 exposure and a change in interest rates occurred during January 2008. Consider an investor that desired synthetic beta exposure for one month (31 days), from the end of December 2007 to the end of January 2008. On 31 December 2007, the S&P 500 Index was at 1468.4, the federal funds rate was 4.6 percent, and the forward dividend yield on the S&P 500 stocks was 2.0 percent. The near-term S&P 500 futures contract had an expiration date of 19 March 2008, or 80 days to expiration, and a futures price of 1477.2. With the then current federal funds rate as the risk-free rate and using Equation C1 in Appendix C, the “fair value” for the futures price of this contract was

$$F_0 = S_0(1 + r_F - d) = 1468.4 \left[1 + 4.6\% \left(\frac{80}{360} \right) - 2.0\% \left(\frac{80}{360} \right) \right] = 1476.9,$$

making the actual futures price $1477.2/1476.9 - 1 = 2$ bps above fair value. The small discrepancy from fair value might reflect the expectation of a change in interest rates during the month, a seasonally lumpy dividend yield, the use of a slightly different interest rate (such as LIBOR) by arbitrageurs, a rounding error in the calculation, or simply the impact of imperfect arbitrage between the futures and spot prices. An investor wanting synthetic beta exposure would have taken a long position in the futures contract at the market price and invested in cash at the federal funds rate. The overnight federal funds rate dropped throughout January 2008 and ended the month at a much lower value (3.1 percent). The result was that the actual return on overnight cash was not enough to make up for the rate implied in the initial futures price.

Specifically, on 31 January 2008, the S&P 500 Index was at 1378.6, and the March futures contract had only 49 days to expiration and a price of 1379.6. With the new lower interest rate of 3.1 percent, the fair-value calculation was

$$F_0 = S_0(1 + r_F - d) = 1378.6 \left[1 + 3.1\% \left(\frac{49}{360} \right) - 2.0\% \left(\frac{49}{360} \right) \right] = 1380.7,$$

so the actual price was $1379.6/1380.7 = 8$ bps *below* fair value. Thus, the synthetic-beta investor underperformed the actual return on the S&P 500 both because the interest rate dropped during the month (leading to a lower return on cash) and because the futures contract happened to go from 2 bps above fair value to 8 bps below fair value. On the basis of the numbers above, the S&P 500 returned $1378.6/1468.4 - 1 = -6.12$ percent before dividends for January, but the direction of the market return is not directly relevant to the issue of tracking error in synthetic exposure. Given the interest rate decline and the drop in the futures price relative to fair value, the tracking error would have been negative even if the S&P 500 had gone up in January 2008. An exact calculation (not shown), with day-to-day interest rates and actual dividends paid, indicates that the synthetic S&P 500 return was a total of 29 bps below the actual S&P 500 return for the month.

Measurement Issues

Chapter 4 contained several examples of beta measurement using *ex post* regressions on return data over a 10-year period. *Ex post* measurement of beta may be acceptable for purposes of portfolio performance attribution but not for the real-time hedging required for alpha-beta separation. In reality, measuring beta for hedging purposes is more complicated than implied by the simple examples in Chapter 4. First, betas used in real time must be assessed *ex ante* rather than *ex post*. Second, beta exposures of an actively managed fund can change over time owing to changes in fund management or changes in the risk structure of the market. Specifically, multifactor betas often contain complex, unstable correlation structures. Third, as explained in Chapter 4, the correct market index (or list of market indices) on which to measure beta (or multifactor betas) for any given fund is inherently subjective.

Although the illustrations in Chapter 4 were based on time-series estimates, a regression analysis of historical fund returns is only one way to estimate the beta exposures in an actively managed portfolio. More accurate beta estimates may be possible through cross-sectional analysis of current fund holdings (i.e., a comprehensive listing of securities and portfolio weights), together with a risk model or estimated covariance matrix. Several commercial risk models, with embedded asset return covariance matrix estimates, are available and in common use among institutional investors. Because portfolio holdings change over time, a risk model for individual securities will arguably give more accurate estimates of fund beta than will time-series analysis, especially if the strategy entails dramatic shifts in exposure. For example, in a tactical asset allocation strategy between stocks and cash, the market beta of the fund varies over time by design. The highs and lows in beta exposure in such a strategy are not captured in a single time-series regression of historical fund returns.

To illustrate alternative beta estimation methodologies and market factor choices, we examine holdings-based beta estimates for the four actively managed equity mutual funds discussed in Chapter 4. Specifically, **Table 6.3** gives estimates of market beta for each fund at the end of December 2007 generated by Barra, a well-known commercial provider of risk estimates for institutional portfolios. The first two rows in Table 6.3 give fund betas with respect to the S&P 500. The first row (“Historical beta” for each fund) provides a weighted average beta of the individual securities, whose security betas are based on 60-month time-series regressions. The “Predicted beta” in the second row is based on Barra’s proprietary risk model, which uses both the fund holdings data and the fundamental risk characteristics of those holdings as estimated by Barra. Although the historical and predicted values are quite close for the first three funds, the difference is material for Fund D, which suggests that recent events have changed its market beta exposure.

Table 6.3. Holdings-Based Beta Estimates

	Fund A	Fund B	Fund C	Fund D
<i>S&P 500</i>				
Historical beta	1.19	0.88	1.11	1.26
Predicted beta	1.15	0.88	0.96	1.01
Size Z-score	-0.38	0.54	0.05	-2.22
Value Z-score	-0.15	-0.05	-0.30	0.17
<i>MSCI World</i>				
Historical beta	0.96	0.72	0.76	0.78
Predicted beta	1.31	0.92	1.15	1.28

The S&P 500 betas shown in Table 6.3 correspond only loosely with the time-series estimates of beta based on 120 months of historical fund returns used as illustrations in Chapter 4 and Table 4.2. There is, however, a clearer correspondence with the Size and Value tilts for each fund. For example, the Barra-estimated Z-scores in Table 6.3 indicate that Fund D has substantial small-cap holdings (the large *negative* value of -2.22 is based on the Barra size factor, which is defined as positive for large-cap holdings), which is consistent with the multifactor regression analysis in Chapter 4 (Table 4.4). In addition, the Value factor Z-scores indicate that Fund C is a growth fund, consistent with the multifactor analysis in Table 4.4.²³ In the last two lines of Table 6.3, we show Barra betas for each fund with

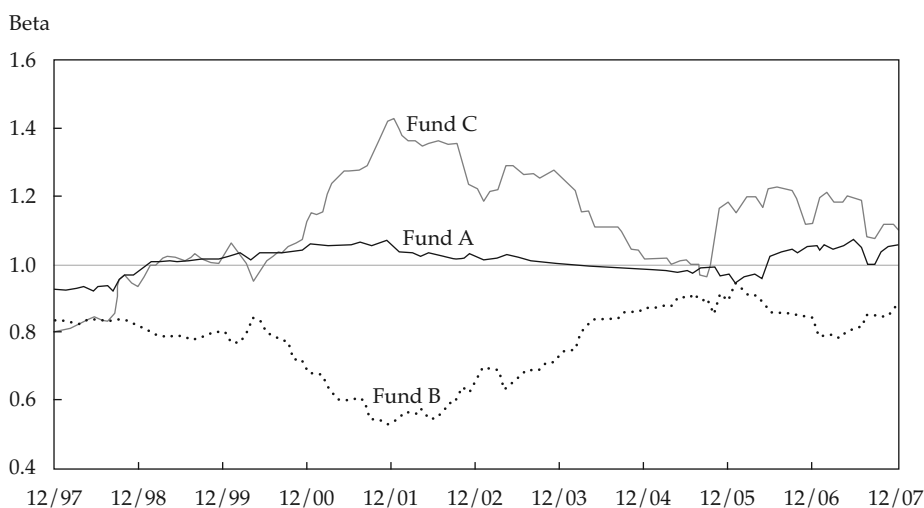
²³The Z-score of an observation in a sample represents the number of standard deviations by which the observation differs from the mean of the sample. When used to measure the exposure of a fund to a factor, the Z-score gives a result that is similar in spirit to, but numerically different from, the factor beta. The factor beta is from a regression on the factor returns calculated as the difference of two market indices, such as Russell 2000 minus Russell 1000 for the small-cap factor.

respect to the global equity market as measured by the MSCI World Index. As mentioned previously, the choice of market factor or factors against which to measure a fund's beta is subjective, driven by the choice of market exposure one wishes to hedge or replicate in alpha–beta separation. Under any choice of market index or estimation methodology, the alpha of a portfolio is defined as a residual return after accounting for one or more beta exposures. Thus, the measurement of a portfolio's alpha, like its beta, is subject to a variety of choices and estimation issues.

The separation of alpha and beta sources of return in an actively managed fund requires an *ex ante* estimate of the beta. The regression coefficients reported in Chapter 4 were “in sample,” meaning that the alphas and betas were estimated by using data that were not available until the end of the period. Regression-based betas calculated in real time are often based on rolling estimates from prior return data. **Figure 6.1** shows the market beta estimated at the beginning of each month for the 10-year period from 1998 to 2007 for the three large-cap domestic equity mutual funds shown in Table 4.2. The beta at each point in time in Figure 6.1 is estimated by using a single-factor regression on the prior 36 months of returns.

The estimated beta for Fund A in Figure 6.1 varies between about 0.9 and 1.1, with an estimation error of about 0.1, for a sample size of 36 months. Given this value, the variation in Fund A's beta over time may be wholly the result of estimation error if we allow for the possibility that the true beta is constant and close to 1.0. But the variation in estimated beta for Fund B ranges from about 0.5 to 0.9—levels

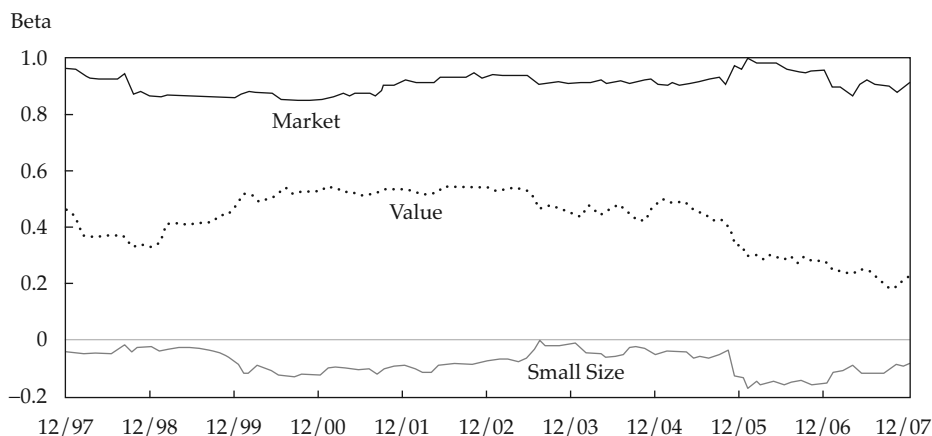
Figure 6.1. Rolling 36-Month Single-Factor Market Betas for Three Mutual Funds



too high to be plausibly explained by estimation error in a single-factor setting. Something in the security selection process or management philosophy of Fund B may have changed over time, or measurement problems may be associated with a single-factor view of the market, as shown subsequently. In any event, the *ex ante* market beta used to hedge market exposure for Fund B was only about 0.5 at the end of 2001 but was as high as 0.9 by 2006. Rising from about 0.8 in 1998 to more than 1.4 by the end of 2001 and then falling, the variation in market beta for Fund C in Figure 6.1 is also higher than could plausibly be explained by estimation error. The key takeaway from this analysis is that hedging and replication require *ex ante*, not *ex post*, estimates, and these estimates can and do change over time.

In Figure 6.1, the relatively low market beta for Fund B in 2001, coincident with a relatively high market beta for Fund C in the same year, suggests that something may have happened in the market at that point in time. The multifactor analysis in Chapter 4 confirmed the conventional wisdom that Fund B (Washington Mutual) is a value-style fund and that Fund C (Janus) is a growth-style fund. Specifically, when a Value factor was introduced into the regression in Table 4.4, the low market beta of Fund B and the high market beta of Fund C were both pulled closer to 1.0. These observations, together with the fact that the Market and Value factors have substantial negative correlation (as shown in Table 4.3), indicate that the market beta variations in Figure 6.1 may have been caused by events in the market, such as the bursting of the tech bubble, rather than by changes in the management style of the funds.

Figure 6.2 plots an *ex ante* three-factor rolling regression for Fund B, the value-style fund. As in Chapter 4, we add Small-Size and Value factors to the Market factor on the basis of differential returns in the various Russell size and style indices. In a multifactor setting, the market beta of Fund B is much closer to 1.0, as was observed in the simple *ex post* analysis in Chapter 4. In addition, Figure 6.2 indicates that the market beta does not vary materially over time, ranging from just under 0.9 to 1.0, in contrast to the single-factor analysis in Figure 6.1. A similar market beta stabilization effect occurs for Fund C (not shown), the growth-style fund. The Small-Size beta for Fund B in Figure 6.2 is slightly negative and varies over time, but not enough to provide evidence that the large-cap bias, if one exists, has changed. The Value factor beta for Fund B is significantly positive and varies around 0.4, but the variations are not large with respect to the estimation error, except for perhaps the drop to 0.2 at the end of the 10-year period. This last fact suggests that the value orientation of Fund B may have changed, although other possibilities exist (e.g., additional missing equity market factors, such as momentum). The key takeaway from this analysis is that the use of multifactor betas to analyze funds can lead to better hedging and replication of market exposures.

Figure 6.2. Rolling 36-Month Multifactor Betas for Fund B

Liquidity Considerations

Liquidity management plays an important role in successfully implementing an alpha–beta separation structure. The distinction we drew between committed capital and contingent capital in Chapter 2 is important in the initial funding of the portfolio structure. Parts of the portfolio will require immediate funding of committed capital but little funding of contingent capital incorporated in derivatives. In addition, adequate liquidity must be planned for in the event that contingent capital is needed for the settlement of losses. Over time, some parts of the portfolio may involve capital calls when commitments have been made to fund new investments. The investor may also need to make periodic withdrawals from the portfolio to spend for other purposes. Although some of these issues are not limited to alpha–beta separation structures, the liquidity issues may be magnified by the use of leverage and derivatives.

Liquidity is also an important consideration when the investor wants to rebalance by altering the allocation within the portfolio after market segments have experienced substantial differential returns. If these periods coincide with constrained periods of market liquidity, withdrawing funds from less liquid strategies to raise cash may be difficult. Indeed, the managers may enforce lockup or withdrawal provisions that limit the investor’s access to part of the funds. The recent poor performance of the markets created liquidity problems for many portfolios (see Siegel 2008). Liquidity considerations are an important part of allocating and managing risk over time. **Exhibit 6.1** presents a checklist of the important implementation issues discussed in this chapter.

Exhibit 6.1. Checklist of Implementation Issues

Portfolio Structure

1. Diversification and risk allocation
2. Degree of true alpha–beta separation
3. Initial funding: committed vs. contingent capital
4. Separate portfolio vs. commingled fund
5. Valuation and pricing procedures
6. Custody of funds and accounting audits
7. Use of leverage
8. Portfolio constraints (e.g., regulatory, legal, investor imposed)

Alpha Management

1. Identified sources of alpha
2. Alpha measurement: pure alpha vs. alpha and partial beta
3. Finding managers with alpha-generating skill
 - a. Reasonable value-added strategies
 - b. Costs (e.g., management fees, trading and administrative costs)
 - c. Capacity constraints
 - d. Alignment of investor and manager incentives
 - e. Historical performance attribution
4. Counterparty risk
5. Performance measurement and attribution

Beta Management

1. Identified sources of beta
2. Beta measurement
3. Choice of implementation vehicles
 - a. Tracking error
 - b. Costs
 - c. Capacity constraints
 - d. Source of funding
4. Counterparty risk
5. Performance measurement and attribution

Liquidity Management

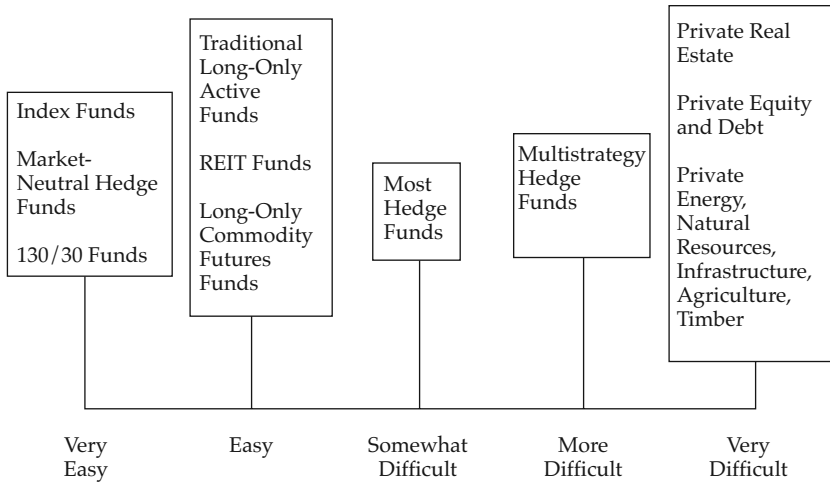
1. Initial funding: committed vs. contingent capital
 2. Settlement of contingent-capital cash flows
 3. Spending needs and deferred capital calls
 4. Rebalancing
 5. Lockup provisions
-

7. Reunion of Alpha and Beta

As shown in Chapter 3, separating the alpha and beta components of an actively managed fund adds value: the freedom to choose different allocations to the beta and alpha sources of expected return. The alpha sources can be optimally weighted in the total portfolio on the basis of their incremental risk–return trade-offs, which may be quite different from the desired allocation to the asset category (beta exposure) from which the alpha is derived. Alpha–beta separation is impossible, however, in some asset categories, and under certain conditions, separation is possible but is not the most cost-efficient solution. In this chapter, we briefly discuss why separation is infeasible in some illiquid asset categories and then cover the cost argument for the reunion of alpha and beta in traditional asset categories.

Siegel, Waring, and Scanlan (2009) recently noted that although alpha–beta separation is always conceptually important, one should not ignore asset categories in which separation is infeasible. For example, private equity funds can provide both high returns and diversification potential in large institutional portfolios, but the illiquid nature of private equity does not allow for alpha–beta separation. The realized returns of any given private equity fund will be partially determined by the general private equity market. So, a “private equity beta” may exist in theory, but at present, no index funds or derivative securities capture this beta. Moreover, the illiquid nature of private equity makes pure-beta instruments unlikely to be introduced in the near future. In contrast, pure-beta instruments might be introduced in other asset categories (e.g., timber) that do not currently have them. **Figure 7.1** illustrates the continuum of difficulty in separating alpha and beta among asset classes.

Other hedge fund strategies may fall into the category of nonseparable alpha and beta because of lack of transparency rather than illiquidity. When the holdings of a specific hedge fund manager are not revealed, the only way to determine beta exposures is an *ex post* regression of fund returns on a large array of potential beta factors, an inexact process that does not provide accurate beta estimates until the fund has been in operation for some time. Whether various hedge fund strategies can be cheaply replicated by some combination of traditional (e.g., public equity) and alternative (e.g., emerging market, volatility) beta factors is an important and ongoing debate (see, for example, Fung and Hsieh 2001; Jaeger 2005). The point is that alternative asset categories can be an important component of institutional portfolios and should not be dismissed simply because they may not be amenable to the straightforward separation of alpha and beta in more liquid and transparent asset classes.

Figure 7.1. Continuum of Difficulty in Separating Alpha and Beta

Source: Adapted from Siegel, Waring, and Scanlan (2009).

We now examine the logic for the reunion of alpha and beta in what are commonly called 130/30 strategies, although we will use the more general “long–short extension” terminology (see Clarke, De Silva, and Saprà 2004). Under certain conditions, the explicit separation of alpha and beta into two portfolios is not the most cost-efficient solution, even in such liquid and transparent asset classes as large-cap U.S. equity. If the active risk of a fund can be adjusted by altering the *magnitude* of the individual securities’ active weights to suit client needs and the fund does not have a no-short-selling constraint, then the added degree of freedom that comes from alpha–beta separation is no longer relevant. In fact, when such implementation issues as shorting costs and collateral requirements are considered, the explicit separation of alpha and beta sources can be costly because of cross-holdings in individual securities (see Jacobs, Levy, and Starer 1998; Jacobs and Levy 2007).

To illustrate, we consider a global tactical asset allocation (GTAA) fund with only three securities, perhaps representing three countries or geographic regions. For simplicity, we assume that the benchmark portfolio is equally weighted and that each of the three securities has identical risk characteristics—each with a standard deviation of $\sigma = 12$ percent and correlation coefficients between each pair of securities of $\rho = 0.5$.²⁴ In the numerical examples that follow, we focus on the *active* weight of each security, defined as the difference between its weight in the actively

²⁴Modeling the securities to be identical in terms of benchmark weight and risk allows us to abstract from the issue of how the expected active return is assigned to each security and yields the simple result that the market beta of each security is 1.

managed portfolio and its benchmark. Unlike total weights that sum to 1, active weights sum to 0 among the securities in the portfolio. Although the technical details are unimportant, the assumptions behind this simple three-asset illustration lead to the result that the magnitude of the three optimal active weights is equal to the ratio of active portfolio risk to security risk: $-\sigma_A/\sigma$, zero, and $+\sigma_A/\sigma$.²⁵ For now, suppose that the desired level of active portfolio risk is $\sigma_A = 4$ percent. To obtain active portfolio risk of 4 percent requires active security weights of $-33\frac{1}{3}$ percent, 0 percent, and $+33\frac{1}{3}$ percent (based on the ratio of active portfolio risk to security risk of $4/12$). Table 7.1 shows the security weights for a pure-alpha (i.e., market-neutral, long-short) fund and a beta-only (i.e., market index) fund, together with the weights in an equivalent actively managed fund. The sum of the security weights is 0 percent for the market-neutral fund but is 100 percent for both the market index fund and the actively managed portfolio. Table 7.1 also shows the sum of the absolute values of the security weights for each portfolio, which is important in understanding collateral requirements.

Table 7.1. Security Weights for Active Risk of 4 Percent in a Beta = 1.0 Portfolio

Security	Market Neutral	Index	Managed Fund
1	-33 1/3%	33 1/3%	0%
2	0	33 1/3	33 1/3
3	<u>33 1/3</u>	<u>33 1/3</u>	<u>66 2/3</u>
Total	0%	100%	100%
Sum of absolute values	66 2/3%	100%	100%

The weight of each security in the actively managed fund is simply the sum of its weights in the market-neutral and index funds. Specifically, the actively managed fund is one-third invested in Security 2 and two-thirds invested in Security 3 (i.e., it has less than the index weight in Security 1 and more than the index weight in

²⁵When a set of securities has identical risk parameters— σ^2 for security variance and ρ for pairwise security correlations—the optimal *active* security weights are proportional to a set of zero-mean unit-variance Z -scores, $\Delta w_i = \frac{\sigma_A}{\sigma\sqrt{1-\rho}\sqrt{N}} z_i$, where σ_A is the desired level of active portfolio risk and N is the number of benchmark securities (see Clarke, De Silva, Sapra, and Thorley 2008). The only possible set of Z -scores that sums to 0 and has unit variances for three securities is $-\sqrt{3}/2$, 0, and $+\sqrt{3}/2$. For $N = 3$ and the convenient assumption that $\rho = 0.5$, the active weights are $-\sigma_A/\sigma$, 0, and $+\sigma_A/\sigma$. For an equally weighted benchmark portfolio in which securities have identical risk characteristics, Equation A4 in Appendix A reduces to $\sigma_M = \sigma\sqrt{1+\rho(N-1)}/N$. For $N = 3$, $\rho = 0.5$, and $\sigma = 12$ percent, we have a market risk of $\sigma_M = 9.8$ percent.

Security 3). In the absence of shorting and leverage costs, the expected active return (i.e., alpha exposure) and active risk in the active fund are exactly the same as in the combined market-neutral and index funds. One key difference between the two alternatives, however, is that the market-neutral/index fund combination requires shorting equal to $33\frac{1}{3}$ percent of notional portfolio value, but the actively managed fund has no such requirement. The opportunity to offset the $33\frac{1}{3}$ percent *short* position in Security 1 in the market-neutral fund and the $33\frac{1}{3}$ percent *long* position in Security 1 in the index fund cannot be exploited given the explicit separation of alpha and beta into two funds. The amount of shorting in a strategy matters because of the extra costs associated with short positions. The shorting cost, or “haircut,” is the difference between a market-determined interest rate benchmark (e.g., LIBOR) and the slightly lower rate paid by the broker on short-sale proceeds. This haircut spread varies from security to security, depending on the difficulty the broker has in finding shares to lend, although a rough estimate for fairly liquid equity securities might be 30 bps. Given this estimate, the total portfolio shorting costs of the market-neutral/index fund combination in Table 7.1 is $33\frac{1}{3}$ percent \times 0.003 = 10 bps, as compared with 0 bp for the single actively managed fund, in which the offsetting long and short positions in Security 1 are combined.

Suppose we increase the active risk (i.e., alpha exposure) in our simple numerical example to 6 percent, which gives a ratio of active portfolio risk to security risk of 6/12. The three active security weights are now -50 percent, 0 percent, and $+50$ percent. Table 7.2 shows the security weights for the market-neutral and index funds, together with the equivalent actively managed fund, which is now a long-short extension fund with a long-short ratio of about 117/17. Although the active fund has short positions, the fund is 100 percent *net-long* and thus still has a market beta of 1. At 6 percent active risk, the single active fund incurs shorting costs on $16\frac{2}{3}$ percent of its notional value, which is much lower (and thus more cost-effective) than the 50 percent short position in the market-neutral fund.

Table 7.2. Security Weights for Active Risk of 6 Percent in a Beta = 1.0 Portfolio

Security	Market Neutral	Index	Managed Fund
1	-50%	33 $\frac{1}{3}$ %	-16 $\frac{2}{3}$ %
2	0	33 $\frac{1}{3}$	33 $\frac{1}{3}$
3	<u>50</u>	<u>33 $\frac{1}{3}$</u>	<u>83 $\frac{1}{3}$</u>
Total	0%	100%	100%
Sum of absolute values	100%	100%	100%

In addition to shorting costs, capital constraints and associated leverage costs are a potential disadvantage of separate funds for alpha and beta. To illustrate, we consider the margin requirements under the assumptions that Regulation T (initial margin of 50 percent, or maximum leverage of 2-to-1) applies to the strategy in Table 7.2 and that we have \$100 million to invest. Regulation T may or may not be relevant in many institutional settings, but the well-known 2-to-1 leverage threshold provides a useful reference point. We choose \$100 million in capital for our illustration so that the percentage weight numbers in the tables can be interpreted as millions of dollars. For example, in the market-neutral fund in Table 7.2, the minimum margin requirement under Regulation T is half the absolute value of the exposures, or \$50 million. This amount of collateral allows the investor to sell \$50 million short of Security 1 and invest \$50 million in Security 3. The minimum collateral required for the \$100 million exposure in the index fund is also \$50 million; thus, the investor has just enough capital—\$100 million—to fund the market-neutral fund and the index fund separately.

Alternatively, the investor can use the single actively managed fund in Table 7.2 to achieve the same active management objective. Specifically, one would short-sell $\$16\frac{2}{3}$ million of Security 1 and invest $\$33\frac{1}{3}$ million in Security 2 and $\$83\frac{1}{3}$ million in Security 3, for a total absolute exposure of $\$133\frac{1}{3}$ million. The minimum capital required under Regulation T for the managed fund is half the sum of the absolute exposures, or $\$66\frac{2}{3}$ million. In other words, all \$100 million of the investor's capital is required to meet the minimum collateral requirement for the market-neutral/index fund combination in Table 7.2 but not for the equivalent long-short extension fund.

The initial margin requirements under Regulation T cannot be met if we increase the level of active risk (i.e., alpha exposure) to an even higher level—8 percent. As shown in Table 7.3, at this level of active risk, the market-neutral security weights (and millions of dollars of security exposure) are $-66\frac{2}{3}$ percent, 0 percent, and $+66\frac{2}{3}$ percent. Assuming that we still need \$50 million in capital for the 2-to-1 leveraged investment in the index fund, the remaining \$50 million in capital must be leveraged $133.33/50$, or 2.67-to-1, in the market-neutral fund, which exceeds the limit set by Regulation T. But the leverage of what is now a $133/33$ long-short extension managed fund in the last column of Table 7.3 is only $166.67/100$, or 1.67-to-1. Although conforming to Regulation T may not be required, higher leverage inherently entails increased operational issues associated with margin calls and marking-to-market. In addition, the cost of borrowing, expressed as the interest rate charged to borrow cash from the broker, increases with fund leverage because of the increased risk of default. The cost of leverage and the haircut cost of shorting are both included in the full “debit-credit” spread—the difference between the interest rate paid to leverage long positions (e.g., LIBOR *plus* 20 bps) and the interest rate paid on short-sale proceeds (e.g., LIBOR *minus* 30 bps).

Table 7.3. Security Weights for Active Risk of 8 Percent in a Beta = 1.0 Portfolio

Security	Market Neutral	Index	Managed Fund
1	-66 2/3%	33 1/3%	-33 1/3%
2	0	33 1/3	33 1/3
3	<u>66 2/3</u>	<u>33 1/3</u>	<u>100</u>
Total	0%	100%	100%
Sum of absolute values	133 1/3%	100%	166 2/3%

The relative contribution of alpha versus beta sources of return in the previous examples was adjusted by increasing the active risk of 100 percent net-long portfolios with a market beta of 1. We can also change the relative magnitude of the alpha and beta sources in an actively managed fund, however, by decreasing the market beta while holding the active risk fixed. For example, consider the medium (6 percent) active risk example in Table 7.2 but with a reduced dollar exposure to the index fund, as shown in Table 7.4. The actively managed portfolio in the last column of Table 7.4 is similar to the one in Table 7.2 (both have an active risk of 6 percent) but is no longer constrained to be 100 percent net-long. Because each of the securities in our example has a market beta of 1.0, the \$90 million in long exposures minus the \$30 million in short exposures results in a portfolio beta of 0.6 when calculated on the capital commitment of \$100 million. Extrapolating on the common vernacular used for long-short extension funds, in which the long and short exposures are divided by the capital commitment, the managed portfolio in Table 7.4 would be called a “90/30 fund.”

Given the variety of possible fund types (130/30, 90/30), a better way to characterize the relative contributions to alpha and beta sources of return is with a risk budget (see Equation A15 in Appendix A), similar to our *ex post* analysis of mutual fund and hedge fund risks in Chapter 4. For example, the first line in Table 7.5 describes the long-only (i.e., 100/0) managed fund in Table 7.1, with a relatively low

Table 7.4. Security Weights for Active Risk of 6 Percent in a Beta = 0.6 Portfolio

Security	Market Neutral	Index	Actively Managed Fund
1	-50%	20%	-30%
2	0	20	20
3	<u>50</u>	<u>20</u>	<u>70</u>
Total	0%	60%	60%
Sum of absolute values	100%	60%	120%

Table 7.5. Alpha and Beta Contribution by Fund Type

Fund Type	Alpha Risk	Beta	Beta Risk	Risk Budget		
				Alpha	Beta	Total
100/0	4.0%	1.0	9.8%	14%	86%	100%
117/17	6.0	1.0	9.8	27	73	100
133/33	8.0	1.0	9.8	40	60	100
90/30	6.0	0.6	5.9	51	49	100
70/40	6.0	0.3	2.9	81	19	100

active risk of 4.0 percent. Given the market beta of 1.0 and market risk of 9.8 percent, Table 7.5 indicates that the alpha exposure of this fund contributes 14 percent of the total portfolio volatility and that beta exposure contributes the remaining 86 percent. As active risk increases in the 117/17 and then the 133/33 managed funds, the alpha source increases to 27 percent and then to 40 percent of the total risk budget, as shown in the second and third rows of Table 7.5. For a constant active risk of 6 percent, the contribution of alpha risk to total risk increases from 27 percent for the 117/17 (beta = 1.0) long-short extension fund in Table 7.2 to 51 percent for the 90/30 (beta = 0.6) fund in Table 7.4 and up to 81 percent for the 70/40 (beta = 0.3) fund. Not shown in a separate table, the beta = 0.3 active fund is similar to Table 7.4, with index fund beta exposures of \$10 million instead of \$20 million in each security.

In summary, the relative contributions of alpha and beta to total volatility in an actively managed fund can be tailored to a client's preferences by adjusting either the magnitude of the active weights (increasing alpha exposure) or the beta exposure. In either case, an essential characteristic of the fund is the absence of a short-selling constraint. The ability to short individual securities allows the contribution of the alpha source to be adjusted without any loss of implementation efficiency as measured by the transfer coefficient, defined by Clarke, De Silva, and Thorley (2002) as the correlation between expected security returns and their constrained active weights. One intuitive way to think about the transfer coefficient is as the ratio of the information ratio for a given portfolio to the information ratio for a hypothetical unconstrained portfolio that is otherwise identical. The transfer coefficient thus represents the proportion of information gathered that is not thrown away because of the existence of the short-selling constraint (see Grinold and Kahn 2000). If short selling is constrained and the transfer coefficient is reduced, the loss in expected alpha may be greater than the shorting and leverage cost savings from combining alpha and beta sources into one fund.

This simple three-security example shows that the separation of alpha and beta sources of return into funds serviced by different brokers or managers may result in less efficient use of security cross-holdings and, therefore, unnecessary shorting and leverage costs. More realistic examples using many securities and market-cap-weighted benchmarks involve mathematics covered in a paper on

long–short extension portfolios by Clarke, De Silva, Sapra, and Thorley (2008). Under the assumption that all the securities in an N -security benchmark have the same risk parameters, the pre-cost expected level of shorting in a market-neutral portfolio, S_{MN} , is

$$S_{MN} = \frac{\sigma_A \sqrt{N}}{\sigma \sqrt{1-\rho} \sqrt{2\pi}}, \quad (7.1)$$

where σ_A is the level of active portfolio risk, σ and ρ are security risk parameters, and N is the number of securities. The expected amount of shorting needed for the same level of active risk (i.e., alpha exposure) in a long–short extension portfolio, S_{LS} , is approximately

$$S_{LS} \approx S_{MN} (1 - N_E/N), \quad (7.2)$$

where N_E is the “effective N ” of the benchmark. Effective N is the number of equally weighted securities that, if held, would have the same diversification impact as the cap-weighted securities actually held in the benchmark; this number is a useful measure of benchmark concentration (see Strongin, Petsch, and Sharenow 2000). For example, as of year-end 2006, the S&P 500 benchmark had an effective N of about $N_E = 125$. For any given level of active risk, the approximation in Equation 7.2 indicates that an S&P 500–benchmarked long–short extension portfolio whose security cross-holdings are exploited has about $125/500 = 25$ percent less shorting than an equivalent market-neutral/index fund combination.

Besides the reduction in shorting costs, lower aggregate levels of shorting lead to lower leverage and thus fewer regulatory or other institutional constraints on capital. If S represents the amount of shorting as a percentage of notional portfolio value, the leverage of either a market-neutral or a long–short extension portfolio (i.e., either S_{MN} in Equation 7.1 or S_{LS} in Equation 7.2) is

$$L = \frac{1+2S}{1}. \quad (7.3)$$

For example, if the amount of shorting in a market-neutral portfolio is $S_{MN} = 60$ percent, the overall portfolio leverage of the market-neutral/index fund combination is $1 + 2 \times 0.60 = 2.2/1$. But according to Equation 7.2, the equivalent S&P 500 long–short extension fund will have only $S_{LS} = (1 - 0.25) \times 60$ percent = 45 percent short positions and thus an overall portfolio leverage of $1 + 2 \times 0.45 = 1.9/1$. Thus, the market-neutral portfolio exceeds the Regulation T capital constraint, but the equivalent long–short extension does not. More to the point, the extra leverage of the market-neutral portfolio will likely incur higher borrowing costs.

8. Conclusion

The ideas we have presented in this monograph are not new but are merely a collection of a variety of widely discussed perspectives on alpha–beta separation. Possible exceptions include our emphasis on contingent versus committed capital, the formal alpha–beta fund separation theorem, and the focus on long–short extension strategies as a resolution to the costs associated with security cross-holdings. By collecting into one monograph current thoughts on alpha–beta separation from a variety of sources, we have provided a “users’ guide” that is accessible to investment professionals wishing to increase their understanding of this important development in institutional portfolio management.

We have argued for the formal separation of alpha and beta by suggesting that it leads to better reward-to-risk outcomes (and have included mathematical proofs in the Appendices). But like most issues in investment management, alpha–beta separation is more art than science. Even the factor list of beta sources of return in financial markets is subjective, let alone the measurement of how much beta exposure to these factors actually exists in any given portfolio. Consistent alpha generation remains an art in competitive financial markets: It is hard for artists/managers to produce and even harder for art critics/plan sponsors to identify in advance. Moreover, synthetic beta management that uses derivative securities requires trading expertise well beyond the simple application of the spot–futures parity relationship.

Are alpha and beta simply the latest buzzwords in investment management practice, or are they fundamentally important concepts? Perhaps the words have started to be overused, but we believe the separation concept is here to stay. Performance measurement and management fee implications alone make drawing a clear distinction between alpha and beta sources of return important. With the advent and wide acceptance of index derivatives in institutional portfolio management, the literal separation of alpha and beta returns has generated a new approach to portfolio management. The practice of alpha–beta separation is likely to accelerate important institutional trends already in motion. These trends include, but are not limited to, the polarization between low-cost beta and high-fee alpha-only product providers, changes in the balance of active versus passive management in the more liquid capital markets, the inclusion of alternative asset classes in institutional portfolios, and the use of risk budgeting in portfolio construction.

Indeed, a risk-budgeting perspective for large institutional portfolio construction may be the most important outcome of explicit alpha–beta separation. Traditional policy debates among investment committees and fiduciaries over asset allocation—focusing on asset-class weights (i.e., variations on the well-worn 65/35 equity/fixed-income rule)—may be replaced with more fruitful discussions about what portion of one’s risk budget to allocate to alpha and what portion to allocate to beta. Once the overall risk budget is established, optimal beta portfolio discussions, on the one hand, may be reminiscent of traditional asset allocation debates, but with a heightened sensitivity to estimated market risk premiums and to correlations among asset betas. Alpha portfolio discussions, on the other hand, will be largely devoid of concern about market risks and risk premiums and will focus instead on credible sources of true alpha wherever and whenever they can be identified. Optimal alpha portfolio debates will also feature discussions of such strategic competitive advantages as leverage, relaxation of the long-only constraint, managerial track records, information ratios, and the appropriateness of fee structures.

In closing, we repeat some of the key concepts of the alpha–beta separation that we covered in this monograph. The separation of the alpha and beta sources of return in institutional portfolios generally leads to a better overall risk–return trade-off than can be achieved by a collection of traditional long-only funds. Alpha is not simply a relative performance perspective on portfolio management; it is pertinent to all market participants, given the existence of low-cost market futures, ETFs, and other forms of index exposure. Alpha–beta separation can be achieved by a derivatives overlay on traditional active managers or by the managers themselves in market-neutral products. The optimal hedge ratio in a derivatives-overlay strategy and the optimal allocation to separate alpha and beta funds lead to the same result: an overall portfolio with an improved Sharpe ratio.

Because alpha and beta risks are generally uncorrelated, the optimal weights of pure-alpha funds are independent of the weights chosen for beta or index funds. The potential improvement in the risk–return trade-off from separating alpha and beta is measured by the squared information ratio of the alpha source. The cumulative impact of several optimally weighted alpha sources can substantially increase the Sharpe ratio of the overall portfolio. Both hedging and replication of the beta source of returns in an actively managed fund require an estimate of the fund’s beta. Funds can have multiple beta exposures, but the correct list of beta factors in any given asset class or fund remains somewhat subjective. The hedging and replication required in the literal separation of alpha from beta returns limit the list of practical beta factors to tradable market indices and associated derivative securities. Beta sources of return extend beyond U.S. equity and fixed-income markets to international capital markets, currencies, commodities, real estate, and any other market that has a tradable contract.

True alpha (positive in realization and uncorrelated with beta) is rare and expensive. In comparison, beta is plentiful and cheap, but synthetic beta returns are not entirely free of cost or devoid of such management issues as tracking error. Beta exposures in real-time hedging and replication must be assessed *ex ante* rather than *ex post*, and they can change over time because of changes in fund management philosophy or changes in the risk structure of the market. The infeasibility of literal alpha–beta separation in less traditional asset classes (e.g., private equity) should not exclude such asset classes from consideration in a large institutional portfolio. But the cost savings from eliminating security cross-listing through the reunion of alpha and beta motivate long–short extensions and other related strategies in traditional asset classes that allow for a customized mix of alpha and beta in a single product. In summary, alpha–beta separation reminds the investor that in the complex world of active fund management, the sum of the parts is often greater than the whole.

Appendix A. Portfolio Risk and Return

The expected return and risk of a multiasset portfolio can be derived from the expected return and risk of the individual assets (e.g., individual funds) by using well-known principles of probability and statistics. The expected portfolio return, $E(r_p)$, is simply the weighted average expected return on the individual assets:

$$E(r_p) = \sum_{i=1}^N w_i E(r_i), \quad (\text{A1})$$

where $E(r_i)$ is the expected return on the i th out of N assets in the portfolio and w_i is the weight of the i th asset. For example, with just two risky assets, A and B , the expected portfolio return is

$$E(r_p) = w_A E(r_A) + w_B E(r_B), \quad (\text{A2})$$

where the weights sum to 1 ($w_A + w_B = 1$). For those familiar with vector-matrix notation, the expected return on the N -asset portfolio in Equation A1 can be expressed in a more compact fashion as

$$E(r_p) = \mathbf{w}'\boldsymbol{\mu}, \quad (\text{A3})$$

where $\boldsymbol{\mu}$ is an $N \times 1$ vector of expected asset returns, \mathbf{w} is an $N \times 1$ vector of asset weights, and $'$ denotes the transpose function.

Portfolio risk as measured by the *ex ante* return variance (i.e., standard deviation squared) is *not* simply an average of the individual asset variances. The formula for portfolio variance, σ_p^2 , depends on the individual asset covariances,

$$\sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{cov}(r_i, r_j), \quad (\text{A4})$$

where the covariance between two assets, $\text{cov}(r_i, r_j)$, can be expressed as the product of the asset standard deviations, σ_i and σ_j , and their correlation coefficient, ρ_{ij} :

$$\text{cov}(r_i, r_j) = \sigma_i \sigma_j \rho_{ij}. \quad (\text{A5})$$

By definition, the correlation of an asset with itself is 1; so, the covariance term in Equation A5 when $i = j$ is simply the individual asset variance, σ_i^2 . For a portfolio of two assets, A and B , Equations A4 and A5 give

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}. \quad (\text{A6})$$

The principle of portfolio diversification applies so long as the individual assets are not perfectly correlated, with more diversification benefit for lower correlations. For example, if two individual assets have the same risk and the correlation coefficient is zero, an equally weighted portfolio in Equation A6 has a standard deviation that is only about 71 percent (i.e., $\sqrt{1/2}$) of that of the individual assets.

By using vector-matrix notation, the N -asset portfolio variance in Equation A4 can be expressed in a more compact fashion as

$$\sigma_P^2 = \mathbf{w}' \mathbf{\Omega} \mathbf{w}, \quad (\text{A7})$$

where $\mathbf{\Omega}$ is an $N \times N$ asset covariance matrix. The matrix equivalent of the two-asset covariance decomposition in Equation A5 yields a formula for portfolio variance that focuses on asset correlations. Let $\boldsymbol{\sigma}$ be an $N \times 1$ vector of individual asset standard deviations. Portfolio variance can then be expressed as

$$\sigma_P^2 = (\mathbf{w} \cdot \boldsymbol{\sigma})' \mathbf{\Pi} (\mathbf{w} \cdot \boldsymbol{\sigma}), \quad (\text{A8})$$

where $\mathbf{\Pi}$ is an $N \times N$ asset *correlation* matrix, and the dot operator indicates element-wise multiplication.

These equations describe the properties of multiasset portfolios, including the case in which one of the assets is cash that earns a risk-free return. The risk-free asset (e.g., cash), however, has unique properties within a risky portfolio that are worth special consideration, especially the linear scaling of portfolio excess return and risk. Consider the expected return of a two-asset portfolio in Equation A2, where asset A is the risk-free asset with return r_F , and asset B is some risky asset or portfolio of risky assets. Acknowledging that the two individual asset weights must sum to 1 and that r_F is certain (i.e., does not need an expectations operator), we have

$$E(r_P) = (1 - w_B)r_F + w_B E(r_B). \quad (\text{A9})$$

The risk of the portfolio, as given in Equation A6, with $\sigma_A = 0$ (because asset A is risk-free), is

$$\sigma_P^2 = w_B^2 \sigma_B^2. \quad (\text{A10})$$

The Sharpe ratio of a portfolio (SR_P), an important measure of risk-adjusted reward, is defined as the expected return in *excess* of the risk-free rate divided by the standard deviation of the portfolio return:

$$SR_P = \frac{E(r_P) - r_F}{\sigma_P}. \quad (\text{A11})$$

Substituting Equations A9 and A10 for a risky portfolio with cash into the Sharpe ratio definition in Equation A11 gives

$$SR_P = \frac{E(r_B) - r_F}{\sigma_B}, \quad (\text{A12})$$

where, notably, the weights cancel out. In other words, modifying the amount of cash in a risky portfolio does not affect the Sharpe ratio because cash exerts a proportional influence on both the excess return (numerator) and the standard deviation (denominator). Similarly, borrowing cash to leverage a portfolio (i.e., a negative weight on cash) increases the risk and excess return of a portfolio but does not affect its Sharpe ratio.

We can decompose a portfolio's return and risk into the contribution from each asset on the basis of a framework noted by Litterman (1996). The contribution of asset i to the total portfolio expected excess return, CR_i , is a fairly straightforward extension of Equation A1:

$$CR_i = w_i [E(r_i) - r_F]. \quad (\text{A13})$$

The contribution of asset i to the total portfolio variance, CV_i , a process sometimes referred to as "risk budgeting," is a bit more involved. Using the definition for two-asset covariance in Equation A5, we reorder the elements in Equation A4 as

$$\sigma_P^2 = \sum_{i=1}^N w_i^2 \sigma_i^2 + 2 \sum_{i=1}^N \sum_{j \neq i}^N w_i w_j \sigma_i \sigma_j \rho_{ij}. \quad (\text{A14})$$

If half of the pairwise covariance contribution (i.e., the second term on the right-hand side of Equation A14) is allocated to each asset in the pair, we can specify the contribution to variance of asset i as

$$CV_i = w_i^2 \sigma_i^2 + \sum_{j \neq i}^N w_i w_j \sigma_i \sigma_j \rho_{ij}, \quad (\text{A15})$$

so that the sum of CV_i among all N assets completely decomposes (i.e., sums to) total portfolio variance, σ_P^2 .

Appendix B. Portfolio Optimization

Mean–variance portfolio optimization is a normative (i.e., prescriptive) theory of how the weights on individual assets in a portfolio *should* be chosen in the trade-off between risk and expected return. Although there are other theories of investor choice (e.g., von Neumann–Morgenstern utility), mean–variance is the most widely accepted optimization process among practitioners. Mean–variance optimization does *not* depend on the validity of the CAPM or any other positive (i.e., descriptive or predictive) theory of asset returns. Optimization is typically conducted by a numerical search routine or by closed-form derivative calculus for unconstrained portfolios. The objective function is to maximize the investor’s mean–variance utility, defined as the expected portfolio return minus portfolio variance and scaled by a risk-aversion parameter, λ ,

$$U = E(r_p) - \frac{1}{2} \lambda \sigma_p^2, \quad (\text{B1})$$

subject to the budget restriction that the weights sum to 1. Using the matrix formulations in Equations A3 and A7, the optimization problem is to maximize

$$U = \boldsymbol{\mu}' \mathbf{w} - \frac{1}{2} \lambda \mathbf{w}' \boldsymbol{\Omega} \mathbf{w}. \quad (\text{B2})$$

Using calculus, the solution (i.e., the vector of optimal weights) to the optimization problem in Equation B2 is

$$\mathbf{w}^* = \frac{1}{\lambda} \boldsymbol{\Omega}^{-1} (\boldsymbol{\mu} - r_Z \mathbf{1}), \quad (\text{B3})$$

where the -1 superscript indicates the matrix inverse function, $\mathbf{1}$ is an $N \times 1$ vector of ones, and r_Z is a constant that adjusts the expected asset returns.²⁶

²⁶The return adjustment, r_Z , can be expressed as a function of the expected return and risk of the global minimum variance (GMV) portfolio: $r_Z = E(r_{GMV}) - \lambda \sigma_{GMV}^2$. The global minimum variance portfolio has the lowest possible risk (i.e., at the left-most tip) of all portfolios on the efficient frontier. Calculus applied to the optimization problem for the global minimum variance portfolio gives $1 / \sigma_{GMV}^2 = \mathbf{1}' \boldsymbol{\Omega}^{-1} \mathbf{1}$ and $E(r_{GMV}) / \sigma_{GMV}^2 = \boldsymbol{\mu}' \boldsymbol{\Omega}^{-1} \mathbf{1}$.

The basic intuition of Equation B3 is that optimal asset weights increase with higher expected returns and decrease with higher variance. For example, in the special case of a diagonal covariance matrix (i.e., zero correlation between all asset pairs), the optimal weight for each asset in Equation B3 reduces to

$$w_i^* = \frac{1}{\lambda} \frac{E(r_i) - r_Z}{\sigma_i^2}. \quad (\text{B4})$$

Standard presentations of portfolio theory include the efficient frontier curve that plots the positions of risky-asset portfolios described by Equation B3 for various levels of risk aversion or, alternatively, various values for the expected portfolio return. The well-known “fund separation theorem” (Tobin 1958) states that when risk-free cash is included in the investment set, the only efficient frontier portfolio of interest is the tangency portfolio. The tangency portfolio is the efficient frontier portfolio that has the maximum Sharpe ratio (geometrically, the slope of a straight line from that portfolio to the risk-free rate) as defined in Equation A11. With the perspective from Appendix A that cash simply scales the risk and expected excess return of a portfolio, one can bypass much of the interim mathematics in standard presentations of portfolio theory by finding the unique risky-asset portfolio with the maximum Sharpe ratio. The formal optimization problem is to maximize

$$SR = \frac{E(r_P) - r_F}{\sigma_P}, \quad (\text{B5})$$

subject to the budget restriction that the optimal weights sum to 1. Using calculus, we find that the solution to this optimization problem is

$$\mathbf{w}^* = \frac{1}{c} \mathbf{\Omega}^{-1} (\boldsymbol{\mu} - r_F \mathbf{1}), \quad (\text{B6})$$

where c is a scaling factor.²⁷ The basic intuition of Equation B6 is that optimal risky-asset portfolio weights increase with higher expected excess (i.e., net of the risk-free rate) returns and decrease with higher variance. For example, in the special case of a diagonal covariance matrix (i.e., zero correlations between all assets), the optimal weight for each risky asset in Equation B6 is

$$w_i^* = \frac{1}{c} \frac{E(r_i) - r_F}{\sigma_i^2}. \quad (\text{B7})$$

²⁷The scaling factor can be expressed as a function of the expected excess return and variance of the global minimum variance portfolio (see Footnote 26): $c = [E(r_{GMV}) - r_F] / \sigma_{GMV}^2$. As a practical matter, one can simply calculate relative weight values in Equation B6 without the scaling factor and then scale them by their sum so that the weights add to 1.

If we multiply each side of Equation B7 by $w_i^* \sigma_i^2$, we find that

$$(w_i^* \sigma_i)^2 = \frac{1}{c} w_i^* [E(r_i) - r_F], \quad (\text{B8})$$

which states that the contribution of asset i to portfolio variance (Equation A15) is proportional to its contribution to portfolio expected excess return (Equation A13). Although Equation B8 is a special case (i.e., diagonal covariance matrix), it illustrates an important general property of optimal active weights. For a portfolio to be optimally weighted, the proportional contribution to portfolio risk for each asset i must be equal to its proportional contribution to portfolio excess return.

Appendix C. Financial Futures and Hedging

Here, we review the returns, risks, and hedging properties of financial (e.g., S&P 500) futures contracts. Ignoring transaction costs, margin requirements, and the effects of marking-to-market, the *long* position in a financial futures contract gives the holder an expiration date (time t) cash flow equal to the realized spot price minus the beginning-of-period (time 0) futures price: $S_t - F_0$. Alternatively, the time t cash flow to the *short* position is $F_0 - S_t$. The spot-futures parity condition for financial futures requires that the futures price be equal to the current spot price, adjusted for the opportunity cost of capital, or the risk-free rate net of the dividend yield:

$$F_0 = S_0(1 + r_F - d), \quad (C1)$$

where the rates, r_f and d , are measured with respect to the expiration date of the contract (e.g., the annual interest rate of 6.0 percent is entered as 0.5 percent for a one-month contract). The parity condition in Equation C1 is maintained in actual futures markets by the action of arbitrageurs who exploit small deviations. By definition, the total market return based on the notional market value of the futures exposure is the percentage price change plus dividend yield d :

$$r_M = \frac{S_T - S_0}{S_0} + d. \quad (C2)$$

Combining the long futures cash flow, $S_t - F_0$, with Equations C1 and C2 gives $S_0(r_M - r_F)$, or a futures contract “return” of $r_M - r_F$. We put *return* in quotes because the futures contract does not require invested capital but merely requires collateral to ensure settlement of any losses.

Given that the risk-free rate is certain, the expected, as opposed to realized, return to a long futures position is $E(r_M) - r_F$ and the return variance is σ_M^2 —the variance of the underlying asset. Similarly, the expected return on a short futures position is just the opposite: $r_F - E(r_M)$. As with the long position, the variance of the short futures position is σ_M^2 , although the risk exposure in the short position is perfectly *negatively* correlated with the underlying index. The impact of futures positions on the return and risk of a portfolio (e.g., Equations A1 and A4) can be assessed like any other asset, where the market exposure is based on the notional value of the contract. Because derivatives contracts do not require a capital outlay, however, the exposure of the futures position does not contribute to the capital budget restriction that the asset weights sum to 1.

We next analyze the hedging impact of a *short* market index futures position on an actively managed fund. The realized return on a managed fund can be defined as the return on the general market from which the fund selects securities plus an extra “alpha” return that may be positive or negative: $r_M + \alpha$. For managers with above-average skill, we generally assume that the *expected* value of the alpha return, $E(\alpha)$, is positive and that the risk of the alpha return, σ_α , is uncorrelated with the return on the general market. Consider a portfolio consisting of an actively managed fund and a short futures position with a notional value of h for “hedge ratio.” Using these relationships, we find that the return on this hedged portfolio is

$$r_P = r_M + \alpha - h(r_M - r_F). \quad (C3)$$

The expected return on the portfolio in Equation C3 is

$$E(r_P) = (1-h)E(r_M) + E(\alpha) + hr_F, \quad (C4)$$

and the return variance is

$$\sigma_P^2 = (1-h)^2 \sigma_M^2 + \sigma_\alpha^2, \quad (C5)$$

which is based on the assumption that the market and alpha risks are uncorrelated.

We are interested in the *optimal* hedge ratio, defined by the value of h that produces the highest possible Sharpe ratio for the hedged portfolio. As discussed in Appendix B, once a portfolio’s Sharpe ratio is optimized, any desired level of expected return and risk can be obtained by leveraging or deleveraging (i.e., using negative or positive cash positions). Using Equations C4 and C5, we find that the Sharpe ratio for the hedged portfolio is

$$SR_P = \frac{(1-h)[E(r_M) - r_F] + E(\alpha)}{\sqrt{(1-h)^2 \sigma_M^2 + \sigma_\alpha^2}}. \quad (C6)$$

Setting the derivative of Equation C6 with respect to h equal to zero, we find that the optimal hedge ratio is

$$h^* = 1 - \frac{[E(r_M) - r_F] / \sigma_M^2}{E(\alpha) / \sigma_\alpha^2}, \quad (C7)$$

which involves the ratios of expected excess return to variance of the beta and alpha components of the fund.

Note that the optimal hedge ratio in Equation C7 is consistent with the optimal portfolio weights for two uncorrelated risky assets, as given by Equation B7. Consider, for example, a scenario in which the denominator in the second term in Equation C7 happens to be exactly equal to the numerator; thus, the optimal amount of hedging of the managed fund is zero. These same values would yield

equal weights on beta-only (market index) and alpha-only (market-neutral) funds in a two-asset portfolio under Equation B7. In other words, an actively managed fund is equivalent to a two-asset portfolio, with equal weightings on the embedded beta-only and alpha-only funds, ignoring leverage.²⁸ The added value of alpha–beta separation is that, in most cases, the implicit equal weighting of the alpha and beta components of an actively managed fund is suboptimal. For example, if the denominator is twice the value of the numerator in the last term in Equation C7, the optimal hedge is one-half. Ignoring leverage, we find that the equivalent two-asset optimal portfolio described in Equation B7 has weights of 2/3 on the alpha-only fund and 1/3 on the beta-only fund, or *relative* asset weights of 2 to 1.

We can determine the Sharpe ratio of the optimally hedged portfolio by substituting Equation C7 into Equation C6. With some algebra, we find that the Sharpe ratio under optimal hedging is

$$SR_P^* = \sqrt{SR_M^2 + IR^2}, \quad (C8)$$

where SR_M is the Sharpe ratio of the market index and IR is the information ratio of the actively managed fund, defined as $IR \equiv E(\alpha)/\sigma_\alpha$.

The preceding discussion on optimal hedging involved only one actively managed fund and the optimal amounts of alpha and beta exposure in that fund. Most institutional portfolios are composed of many actively managed funds—at least one each for several different asset classes. Optimal simultaneous hedging of several actively managed funds is similar to the general portfolio optimization problem discussed in Appendix B. In that appendix, we referred to the classic “two-fund separation theorem,” which states that in the presence of a risk-free asset (i.e., cash), the optimal mix of risky assets does not vary with the risk aversion (or, alternatively, the expected return requirement) of the investor. Specifically, the relative weights within the optimal risky-asset portfolio are fixed for any given set of investor beliefs, as expressed in the vector of expected asset returns and covariance matrix. The level of overall portfolio expected return and risk can be adjusted by how much cash is combined with this unique optimal risky-asset portfolio.

We now introduce an “alpha–beta fund separation theorem,” which states that the optimal mix of alpha-only funds does not depend on the choice of beta exposures to the various asset classes. This result rests on the assumption that the alpha returns are uncorrelated with the realized return of the market (i.e., beta exposure) from which they are derived or any other beta exposure in a multifund portfolio. This assumption conforms to the notion of “true alpha,” as identified by the statistical

²⁸Because of their lower variances, *exact* risk and return equivalence to an actively managed fund is obtained by 2-to-1 leverage (i.e., –100 percent cash) and allocations of 100 percent each to the beta-only and alpha-only funds.

process of multivariate regression analysis. We also assume that the various alpha returns are uncorrelated with each other, although this is not actually required for the alpha–beta fund separation theorem.

Under these assumptions, the optimal weight on each alpha source is given by Equation B7, which does not include covariance terms. The relative weights or optimal exposures to the various alpha sources depend solely on the expected alpha and active risk of the fund from which they are derived. Specifically, a reformulation of Equation B7 for alpha-only funds states that the optimal weight times active risk is proportional to the fund’s information ratio:

$$w_i^* \sigma_i = \frac{1}{c} IR_i, \quad (\text{C9})$$

where, as in Equation C8, $IR_i \equiv E(\alpha)/\sigma_\alpha$ for the i th fund. In other words, the optimal relative weights (i.e., weights within the alpha-only portfolio) are invariant to the choice of beta-only (i.e., market index) funds. This result holds even if the “policy portfolio” of beta exposures is not chosen optimally with respect to a covariance matrix of market returns. Moreover, when the alphas in each asset class and the active manager are separated from their respective betas and are optimally weighted, the resulting Sharpe ratio for the entire portfolio is an expanded version of Equation C8:

$$SR_P = \sqrt{SR_M^2 + \sum_{i=1}^N IR_i^2}, \quad (\text{C10})$$

where SR_M is the Sharpe ratio for the portfolio of beta-only funds and the series of IR s is the information ratios for N alpha-only funds.

Appendix D. Capital Asset Pricing Model

Although the principles of alpha–beta separation previously described do not depend on the CAPM or other equilibrium models in financial economics, a review of basic CAPM logic is useful. The derivation of the CAPM starts with the decomposition of security returns in the market model of Equation 2.1, and then uses several additional, and perhaps unrealistic, simplifying assumptions. The logic is that (1) ignoring transaction costs, taxes, and other market frictions, (2) assuming all investors have the same beliefs (technically, homogenous expectations), and (3) assuming all investors have the same investment objective (technically, single–period mean–variance utility), then all investors would hold the same portfolio. The only portfolio that all investors can hold simultaneously is the entire market; thus, the only risk that matters in determining the stock price is the portion that cannot be diversified away in the market portfolio, which is measured by the security’s market beta.

The formal CAPM equation, also known as the security market line (SML), states that the expected excess return on any security i is the security’s market beta times the expected excess return on the market:

$$E(r_i) - r_F = \beta_i [E(r_M) - r_F]. \quad (D1)$$

The word “expected” in this context refers to a mathematical expectation or probabilistic average of all the possible returns that might be realized in a given period. Note that Equation D1 follows directly from the market model for security returns in Equation 2.1 by taking the expectation of both sides of the equation and assuming that the expected value of the security alpha is zero. In other words, the CAPM can be thought of as describing the expected return on securities in a perfectly efficient market. In contrast, positive alphas and thus the motive for alpha–beta separation strategies depend on the market being inefficient.

Although the CAPM was intended to be more general, early applications of the theory focused on U.S. equity securities, with the S&P 500 acting as a proxy for the market. A useful conceptual formula from regression analysis for the market beta is

$$\beta_i = \left(\frac{\sigma_i}{\sigma_M} \right) \rho_{iM}, \quad (D2)$$

where σ_i and σ_M are, respectively, the return standard deviations for security and market returns and ρ_{iM} is the correlation coefficient between the security and market returns. Typical numbers for U.S. stocks might be $\sigma_i = 30$ percent, $\sigma_M = 15$ percent, and $\rho_{iM} = 0.5$, which gives a beta of 1.0. Indeed, given that the marketwide return is the cap-weighted average of the individual security returns, linear regression statistics dictate that the cap-weighted average stock have a beta of exactly 1.0. Although the CAPM itself may be an incomplete theory of financial market equilibrium, the use of Equation D2 for calculating market beta exposures and other statistical properties of beta is essential to alpha–beta separation strategies.

References

- Anson, Mark. 2008. "The Beta Continuum: From Classic Beta to Bulk Beta." *Journal of Portfolio Management*, vol. 34, no. 2 (Winter):53–64.
- Arnott, Robert D., Jason Hsu, and Philip Moore. 2005. "Fundamental Indexation." *Financial Analysts Journal*, vol. 61, no. 2 (March/April):83–99.
- Banz, Rolf. 1981. "The Relationship between Return and Market Value of Common Stocks." *Journal of Financial Economics*, vol. 9, no. 1 (March):3–18.
- Basu, Sanjoy. 1977. "Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis." *Journal of Finance*, vol. 32, no. 3 (June):663–682.
- Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance*, vol. 52, no. 1 (March):57–82.
- Clarke, Roger, Harindra de Silva, and Steven Sapra. 2004. "Toward More Information Efficient Portfolios." *Journal of Portfolio Management*, vol. 31, no. 1 (Fall):34–63.
- Clarke, Roger, Harindra de Silva, and Steven Thorley. 2002. "Portfolio Constraints and the Fundamental Law of Active Management." *Financial Analysts Journal*, vol. 58, no. 5 (September/October):48–66.
- . 2006. "Minimum-Variance Portfolios in the U.S. Equity Market." *Journal of Portfolio Management*, vol. 33, no. 1 (Fall):10–24.
- Clarke, Roger, Harindra de Silva, and Brett Wander. 2002. "Risk Allocation versus Asset Allocation." *Journal of Portfolio Management*, vol. 29, no. 1 (Fall):9–30.
- Clarke, Roger, Harindra de Silva, Steven Sapra, and Steven Thorley. 2008. "Long–Short Extensions: How Much Is Enough?" *Financial Analysts Journal*, vol. 64, no. 1 (January/February):16–30.
- Coleman, Thomas S., and Laurence B. Siegel. 1999. "Compensating Fund Managers for Risk-Adjusted Performance." *Journal of Alternative Investments*, vol. 2, no. 3 (Winter):9–15.
- Fama, Eugene F., and Kenneth R. French. 1992. "The Cross-Section of Expected Stock Returns." *Journal of Finance*, vol. 47, no. 2 (June):427–465.

———. 1993. “Common Risk Factors in the Returns on Stocks and Bonds.” *Journal of Financial Economics*, vol. 33, no. 1 (February):3–56.

———. 1996. “Multifactor Explanations of Asset Pricing Anomalies.” *Journal of Finance*, vol. 51, no. 1 (March):55–84.

Fama, Eugene F., and James D. MacBeth. 1973. “Risk, Return, and Equilibrium: Empirical Tests.” *Journal of Political Economy*, vol. 81, no. 3 (May–June):607–636.

French, Kenneth R. 2008. “Presidential Address: The Cost of Active Investing.” *Journal of Finance*, vol. 63, no. 4 (August):1537–1573.

Fung, William, and David Hsieh. 2001. “The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers.” *Review of Financial Studies*, vol. 14, no. 2 (Summer):313–341.

Goodwin, Thomas H. 1998. “The Information Ratio.” *Financial Analysts Journal*, vol. 54, no. 4 (July/August):34–43.

Grinold, Richard C. 1989. “The Fundamental Law of Active Management.” *Journal of Portfolio Management*, vol. 15, no. 3 (Spring):30–37.

Grinold, Richard C., and Ronald N. Kahn. 2000. “Efficiency Gains of Long–Short Investing.” *Financial Analysts Journal*, vol. 56, no. 6 (November/December):40–53.

Haugen, Robert, and Nardin Baker. 1991. “The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios.” *Journal of Portfolio Management* (Spring):35–40.

Jacobs, Bruce I., and Kenneth N. Levy. 2007. “Enhanced Active Equity Portfolios Are Trim Equitized Long–Short Portfolios.” *Journal of Portfolio Management*, vol. 33, no. 4 (Summer):19–25.

Jacobs, Bruce I., Kenneth N. Levy, and David Starer. 1998. “On the Optimality of Long–Short Strategies.” *Financial Analysts Journal*, vol. 54, no. 2 (March/April):40–51.

Jaeger, Lars. 2005. *Through the Alpha Smoke Screens: A Guide to Hedge Fund Returns*. New York: Institutional Investor Books.

Jegadeesh, Narasimhan, and Sheridan Titman. 1993. “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency.” *Journal of Finance*, vol. 48, no. 1 (March):65–91.

Jensen, Michael C. 1968. “The Performance of Mutual Funds in the Period 1945–1964.” *Journal of Finance*, vol. 23, no. 2 (May):389–416.

- Kritzman, Mark. 1998. "Wrong and Alone." *Economics and Portfolio Strategy* (15 January). New York: Peter L. Bernstein, Inc.
- . 2004. "The Insecurity of Security Selectors." *Economics and Portfolio Strategy* (15 April). New York: Peter L. Bernstein, Inc.
- . 2007. "Who Charges More: Hedge Funds or Mutual Funds?" *Economics and Portfolio Strategy* (1 February). New York: Peter L. Bernstein, Inc.
- Kung, Edward, and Larry Pohlman. 2004. "Portable Alpha." *Journal of Portfolio Management*, vol. 30, no. 3 (Spring):78–87.
- Layard-Liesching, Ronald. 2004. "Investing Contingent Capital." *Journal of Portfolio Management* (30th Anniversary Issue):80–87.
- Leibowitz, Martin L. 2005. "Alpha Hunters and Beta Grazers." *Financial Analysts Journal*, vol. 61, no. 5 (September/October):32–39.
- Lintner, John. 1965. "The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics*, vol. 47, no. 1 (February):13–37.
- Litterman, Robert. 1996. "Hot Spots and Hedges." *Journal of Portfolio Management* (Special Issue: December):52–75.
- . 2004. "The Active Risk Puzzle." *Journal of Portfolio Management* (30th Anniversary Issue):88–93.
- Markowitz, Harry. 1952. "Portfolio Selection." *Journal of Finance*, vol. 7, no. 1 (March):77–91.
- Merton, Robert C. 1973. "An Intertemporal Capital Asset Pricing Model." *Econometrica*, vol. 41, no. 5 (September):867–887.
- Mossin, Jan. 1966. "Equilibrium in a Capital Asset Market." *Econometrica*, vol. 34, no. 4 (October):768–783.
- Mulvey, John M., and Woo Chang Kim. 2008. "Active Equity Managers in the U.S." *Journal of Portfolio Management*, vol. 34, no. 2 (Winter):126–134.
- Reinganum, Marc R. 1983. "The Anomalous Stock Market Behavior of Small Firms in January: Empirical Tests for Tax-Loss Selling Effects." *Journal of Financial Economics*, vol. 12, no. 1 (June):89–104.
- Ross, Stephen A. 1976. "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory*, vol. 13, no. 3 (December):341–360.

Sharpe, William F. 1963. "A Simplified Model for Portfolio Analysis." *Management Science*, vol. 9, no. 2 (January):277–293.

———. 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance*, vol. 19, no. 3 (September):425–442.

———. 1991. "The Arithmetic of Active Management." *Financial Analysts Journal*, vol. 47, no. 1 (January/February):7–9.

Siegel, Laurence B. 2008. "Alternatives and Liquidity: Will Spending and Capital Calls Eat Your 'Modern' Portfolio?" *Journal of Portfolio Management*, vol. 35, no. 1 (Fall):103–114.

Siegel, Laurence B., M. Barton Waring, and Matthew Scanlan. 2009. "Five Principles to Hold Onto (Even When Your Boss Says the Opposite)." *Journal of Portfolio Management*, vol. 35, no. 2 (Winter):25–41.

Strongin, Steven, Melanie Petsch, and Gregg Sharenow. 2000. "Beating Benchmarks." *Journal of Portfolio Management*, vol. 26, no. 4 (Summer):11–27.

Tobin, James. 1958. "Liquidity Preference as Behavior toward Risk." *Review of Economics and Statistics*, vol. 25, no. 1 (February):65–86.

Treynor, Jack L. 1962. "Toward a Theory of Market Value of Risky Assets." Unpublished manuscript printed as Chapter 2 in *Asset Pricing and Portfolio Performance*. Edited by Robert A. Korajczyk (1991). London: Risk Books.

Waring, M. Barton, and Laurence B. Siegel. 2003. "The Dimensions of Active Management." *Journal of Portfolio Management*, vol. 29, no. 3 (Spring):35–51.

———. 2006. "The Myth of the Absolute-Return Investor." *Financial Analysts Journal*, vol. 62, no. 2 (March/April):14–21.