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# ***Fundamentals of Futures and Options*** **(a summary)**

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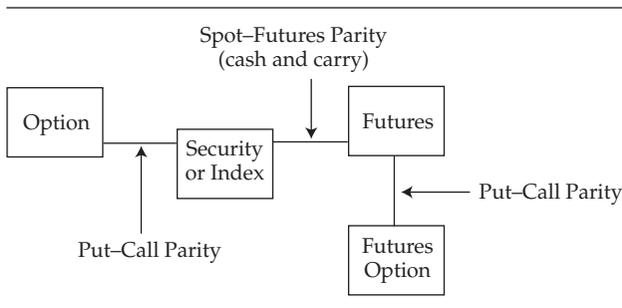
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## **Overview of Derivative Securities and Markets**

The focus of this book is simple financial derivatives—options and futures. The growth of these instruments began in the United States, largely in the 1970s with the organization of the Chicago Board Options Exchange. Futures on U.S. Treasury bonds and notes began trading in the late 1970s, and options on individual stocks and equity indices began trading in the early 1980s. Since then, derivatives have not only expanded to other countries, but the set of underlying indices or assets has also grown. In many cases, the volume of trading in these instruments now exceeds the volume of trading in physical assets. In addition to derivatives of commodities and currencies, derivatives are now traded on a wide array of equity, interest rate, and credit indices; market volatility; inflation; weather; and real estate. Modeling the expected payoffs of many of these contracts requires complex calculations. Such calculations could not have been done without the increased data-handling capabilities and computing power available through modern computers. The growth in the development and use of the more complex derivatives has been an important trend in risk management and investing. In this book, however, we discuss only the relatively simple options and futures contracts. We leave the discussion of more complex derivatives to others.

Options and futures contracts are derivative instruments—that is, they derive their value from some other underlying security or index. The relationships between the underlying security and its associated options and futures contracts are illustrated in **Figure 1**. Note that options may be written on futures contracts but all options and futures ultimately derive their value from an underlying security or index. The links pictured in Figure 1 keep the security and its options and futures coupled together. The arbitrage link between a futures contract and the underlying security is called *spot-futures parity* or *cash-and-carry arbitrage*. The arbitrage linking put and call options to each other is referred to as *put-call parity*, which together with spot-futures parity links the options to the underlying security.

**Figure 1. Arbitrage Links**



Standardized contract features allow futures and options to be traded quickly and efficiently on organized exchanges. The exchanges serve as intermediaries to facilitate trading, to transfer daily gains and losses between parties, and to pool resources of the exchange members to guarantee financial stability if a single investor should default. The individual parties to a given trade may never meet and do not need to deal with each other after an exchange has matched their trade. The exchange’s clearinghouse function allows buyers or sellers to reverse a position before maturity and thus close out the obligation without having to find the party who initially took the other side of the trade.

The use of options and futures gives the investor flexibility in managing the risk of an underlying security or index. Such basic business activities as banking, international trading, and providing retirement benefits may leave an individual investor or corporation exposed to interest rate, foreign exchange, or equity market risk. The use of options and futures allows the investor to hedge or transfer all or some of this risk to others more willing to bear it. Derivative securities can also be used to speculate (assume risk) rather than hedge risk away, although most applications we discuss concern using them for risk control. The book focuses on contracts for financial assets, such as stocks, bonds, and foreign exchange, but structured derivative contracts exist for metals, energy, agricultural, and other physical commodities.

Trading in options and futures contracts has several potential operational advantages over trading in the underlying securities:

- easy adjustment of market exposure,
- reduced transaction costs,
- same-day settlement or simultaneous trades,
- less disruption of underlying asset management, and
- creation of specialized risk/return patterns.

Of course, the use of options and futures contracts also has some disadvantages:

- the need to understand complex relationships,
- potential tracking error to the underlying security or index,
- liquidity reserves required to post and meet margin requirements,
- daily settlement associated with marking to market, and
- potential short-term tax consequences.

## Futures Contracts

A futures contract essentially allows an investor to commit now to the purchase or sale of an underlying asset at a specified price, with delivery and payment delayed until a specified settlement date. No money changes hands up front, except for posting initial margin to reduce the risk of nonpayment, but a futures contract can be either “bought” or “sold.” The buyer of a futures contract has a *long position* and commits to buy the underlying asset or security at the specified price and date. The seller of a futures contract has a *short position* and commits to sell the underlying asset or security at the specified price and date. The fact that the future price for transacting is negotiated now but delivery and payment are delayed until the settlement date creates an *opportunity cost* for the seller in receiving payment. As a result, the negotiated price for future delivery of the asset differs from the current cash price by an amount that reflects the cost of waiting to get paid.

The futures price is related to the price of the underlying security or asset, the interest opportunity cost until expiration, and any expected cash distributions by the underlying asset before expiration. The futures price is tied to the price of the underlying security or index through the arbitrage condition of cash and carry. Because of arbitrage trading activity, when the underlying asset price changes, the price of the futures contract will change accordingly.

One way to think about the use of futures contracts is that the cash-and-carry arbitrage ensures that the futures contract plus a cash reserve will behave like the underlying security:

Futures + Cash ↔ Security.

Specifically, an investor may wish to create the same risk/return profile as the underlying security but use a futures contract because the transaction can be done more quickly and at less cost than buying or selling the underlying security. Such a process can be described as creating a *synthetic security* in place of the actual security.

In addition to creating a synthetic security, one can also rewrite the basic arbitrage relationship to create *synthetic cash*. In fact, creating a synthetic cash position is equivalent to hedging the underlying position in the security:

Security – Futures ↔ Cash.

The cash-and-carry arbitrage relationship keeps the futures contract priced so that an offsetting position relative to the underlying security results in a return to the hedger consistent with a riskless rate. In essence, creating a hedged position eliminates the primary risk in the underlying security by shifting it to others more willing to bear the risk. Of course, the investor's risk could be eliminated directly by simply selling the underlying security position, but this might interfere with the nature of the investor's business or disrupt a continuing investment program. Thus, the futures market provides an alternate way to temporarily offset or eliminate much of the risk in the underlying security position.

## Option Contracts

The two basic types of options are *call* options and *put* options. A call option gives the owner the right to *buy* the underlying security at a specified price within a specified period of time. A put option gives the owner the right to *sell* the security at a specified price within a particular period of time. The *right*, rather than the obligation, to buy or sell the underlying security is what differentiates options from futures contracts.

In addition to buying an option, an investor may also sell a call or put option the investor had not previously purchased, which is often called *writing* an option. Thus, the two basic option positions can be expanded into four option positions, as shown in **Figure 2**. Understanding how put and call option prices behave and how these basic option positions affect an overall portfolio is critical to understanding more complex option strategies.

An exchange-traded option has a price at which it currently trades, sometimes called the option's *premium*. The option premium depends on a number of factors, including the difference between the option contract's *strike* or *exercise price* and the price of the underlying security. Analysts often describe

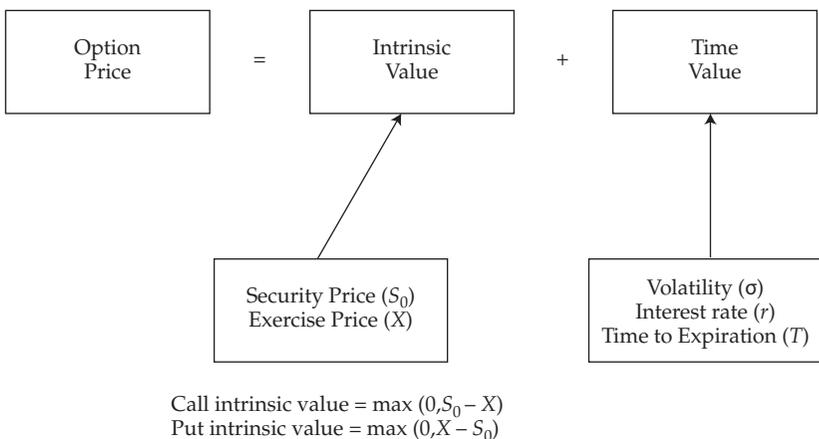
**Figure 2. Option Positions**

	Call Option	Put Option
Buy	Purchased the right to buy the underlying security	Purchased the right to sell the underlying security
Sell or Write	Sold the right to buy the underlying security (might be forced to sell)	Sold the right to sell the underlying security (might be forced to buy)

the option's market price as being composed of two parts—the *intrinsic value* and the *time value*—as illustrated in **Figure 3**. In the case of a call option, the intrinsic value component, also called the *exercise value*, is the amount of money that would be received if an investor were to exercise the option to purchase the underlying security and then immediately sell the security at the current market price. In other words, the intrinsic value depends on the relationship between the current security price,  $S_0$ , and the exercise price of the option,  $X$ . If  $S_0 - X$  is positive, then the call option is *in the money* and has a positive intrinsic value. If  $S_0 - X$  is negative, then the call option is *out of the money* and has zero intrinsic value. Thus, the intrinsic value of a call option is the difference between the security price and the exercise price or zero, whichever is larger. The intrinsic value of a put option is just the reverse: the maximum of  $X - S_0$  or zero, whichever is larger. For a put, the option is in the money if  $X - S_0$  is positive; otherwise, the intrinsic value of the put option is zero.

The second component of the option price, the time value, is the difference or residual between the market price of the option and the current intrinsic value. As shown in Figure 3, the time value component of the option price is a function of the underlying security's expected volatility or risk,  $\sigma$ , the current level of interest rates,  $r$ , and the option's maturity date or time to expiration,  $T$ . The term *time value* comes from the fact that this component of the total option price gradually approaches zero as the option gets close to expiration, leaving only the intrinsic value. The convergence of the option price to the intrinsic value at expiration is similar to the convergence of a futures price to the underlying security price at expiration.

**Figure 3. Option Price Components**



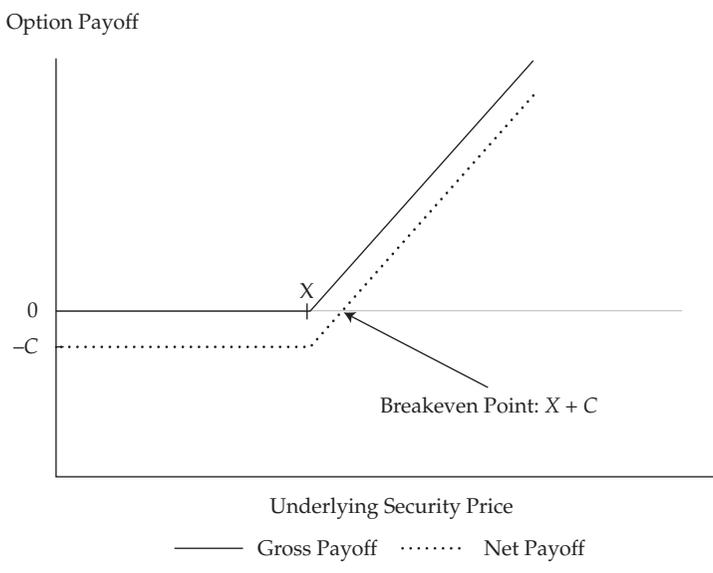
Insight into the characteristics of options can be obtained by examining their payoff values at expiration. A *contingency table* is one technique for showing the expiration value of various option positions and strategies. The contingency table here shows the individual values of a long call, a long put, and the underlying security, contingent on whether the price of the security  $S_T$  is above or below the exercise price on option expiration date  $T$ :

	$S_T < X$	$S_T > X$
Call Option	0	$S_T - X$
Put Option	$X - S_T$	0
Security	$S_T$	$S_T$

As shown in this table, the call option has value at expiration if and only if the underlying asset price is above the strike price,  $X$ , and the put option has value at expiration if and only if the underlying asset price is below the strike price. The value of the underlying security,  $S_T$ , is not contingent on whether it is below or above the option's exercise price.

Another useful tool for option analysis is a *hockey stick* diagram of the option payoff at the expiration date as a function of the underlying asset price. For example, **Figure 4** illustrates the payoff pattern for a call option at expiration. The horizontal axis is the underlying security price, and the

**Figure 4. Payoff Profile of a Call Option**

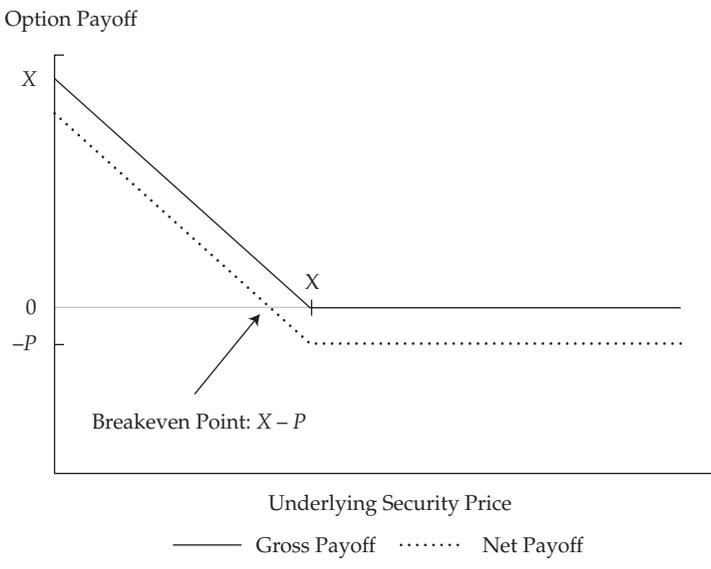


vertical axis measures the gross payoff (solid line) and net payoff (dotted line) of the call option. If the security price ends up below the strike price,  $X$ , the gross payoff to the call option is zero, as shown on the left side of Figure 4. If the security price ends up above the exercise price, the gross payoff to the call option is the difference between the security price and the strike price,  $S_T - X$ , as shown on the right side of Figure 4. Because the investor must pay to initially purchase the option, the diagram includes a net payoff. The net payoff is the gross payoff of the option minus the initial purchase price,  $C_0$ .

The net payoff from the call option is a constant negative value until the security price reaches the exercise price. From that point, the net payoff starts to rise. The investor breaks even, with zero net profit, at the point where the security price equals the strike price plus the initial price paid for the option. Thus, the investor enjoys a positive net profit if the underlying asset price ends up being larger than the strike price plus the initial price paid for the option. Note that the call option payoff diagram has a kinked or asymmetrical payoff pattern, which distinguishes it from a futures contract. This asymmetry in the payoff allows the option to create specialized return patterns at expiration that are unavailable when using a futures contract.

**Figure 5** is the payoff diagram for a put option. The put option has a gross payoff of zero if the underlying security price ends up above the exercise price, as shown on the right side of the diagram. If the underlying asset price is below

**Figure 5. Payoff Profile of a Put Option**



the strike price, the gross payoff to the put option is  $X - S_T$ , with the maximum gross payoff being  $X$  if the underlying security price goes all the way to zero. The net payoff is shown by the dotted line, which is shifted down from the gross payoff by the initial cost of the put option,  $P_0$ . The investor breaks even, with zero net profit, at the point where the security price equals the strike price minus the initial price of the put option. The investor in a put option incurs a net loss if the security price is above that value at the expiration of the option.

The payoff profiles for buying call and put options together with the payoff profiles for selling options can be used to better understand common option strategies. Commonly used strategies include covered calls, protective puts, and more involved option strategies, such as the straddle and the bull call spread.

## Put–Call Parity

The arbitrage relationship for a call and put option pair for the same underlying security with the expiration dates is known as put–call parity. The basic form of the relationship can be written as the spread between the current price of a call option and the current price of the put option, which is equal to the spread between the current security price and a bond equal to the present value of the option strike price:

$$\text{Call} - \text{Put} = \text{Security} - \text{Bond}.$$

If the put–call parity relationship did not hold, one could create greater-than-riskless returns without any risk by selling the expensive combination of assets and buying the cheap combination. Thus, put–call parity is analogous to the cash-and-carry arbitrage condition for futures contracts, also known as spot–futures parity.

Algebraic perturbations of the basic put–call parity relationship can be used to provide insight into synthetic positions. For example, a common form of put–call parity that isolates the current price of the call option is

$$\text{Call} = \text{Security} + \text{Put} - \text{Bond}.$$

In this formulation, the call option can be thought of as being equal to the underlying security plus the corresponding put option minus a cash-equivalent security or riskless bond. In other words, a *synthetic* call option can be created by buying the security and the put option, where most (although not all) of the cost is covered by borrowing the present value of  $X$  dollars. Specifically, the combination of securities on the right side of the equation would produce the same payoff as buying the call option directly.

Another perturbation of the put–call parity relationship,

$$\text{Put} = \text{Call} + \text{Bond} - \text{Security},$$

shows that an investor can create a synthetic put option by buying the call option and investing the present value of  $X$  dollars at the risk-free rate, where most (although not all) of the cost is covered by the proceeds from shorting the underlying security. Again, the combination of securities on the right side would produce the same payoff as buying the put option directly.

In fact, the payoff of the underlying security itself can be replicated by buying a riskless bond with a face value of  $X$ , buying a call option, and writing a put option. In a similar way, a riskless bond can be constructed by buying the underlying security, buying a put option, and selling a call option.

We can also leave *two* terms on each side of the put-call parity formula to create a synthetic covered-call position:

$$\text{Security} - \text{Call} = \text{Bond} - \text{Put}.$$

On the left side of the equation, the traditional covered-call position is established by buying the security and selling the call option, but the right side indicates that an equivalent position is to buy a riskless bond at a discount with a face value of  $X$  and sell the put option.

Finally, a synthetic protective put can be created by buying a riskless bond at a discount with face value  $X$  and buying a call option:

$$\text{Security} + \text{Put} = \text{Bond} + \text{Call}.$$

## Option Pricing

Option prices depend on the price of the underlying security, the strike price of the option, the volatility of the underlying security, the level of interest rates, and the option's time to expiration. Fully understanding the impact of each of these parameters requires a model of the option price, not simply a decomposition of the price into the intrinsic value and time value components. The book reviews the pricing of options through the use of a binomial branching process that allows the investor to price put and call options on the basis of their parameters and the assumed distribution of returns to the underlying security. In the limit, as the branching process is expanded, the binomial pricing logic converges to various forms of the well-known Black-Scholes model.

Specifically, the formula for a European-style call option (an option that can be exercised only on the maturity date) on an underlying asset without cash distributions prior to option expiration is

$$C_0 = S_0 N(d_1) - X e^{-rT} N(d_2),$$

where

$$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

and

$$d_2 = \frac{\ln(S_0 / X) + (r - \sigma^2 / 2)T}{\sigma \sqrt{T}}.$$

The Black–Scholes formula uses a number of functions associated with *continuous time* financial mathematics. For example, the function  $N(\cdot)$  is the cumulative standard normal distribution and the function  $\ln(\cdot)$  is the natural logarithm, with  $e^{(\cdot)}$  as the corresponding exponentiation function. In continuous-time mathematics, present values are calculated by the term  $e^{-rT}$  instead of the more familiar  $1/(1 + R)^T$  formulation used in discrete-time mathematics.

Option analysts use a series of Greek letters to describe how a call option price changes as the various parameters change. The most important “greek” is *delta*,  $\Delta$ , which describes the change in the option price as a result of a change in the price of the underlying security. The delta of an option is also known as the *hedge ratio* because it specifies the number of shares in the underlying security needed to offset a change in option value resulting from changes in the underlying security price. Another greek is *theta*,  $\theta$ , the change in option price arising from the passage of time. Other greeks that are used are also discussed in the book.

The existence of option contracts across a range of strike prices for a single underlying security allows option analysts to measure aspects of the probability distribution for the underlying security’s return that are not directly observable without options markets. Similarly, option contracts with different expiration dates can be used to forecast how the volatility of the underlying security is expected to evolve over time. These advanced probability-distribution and term-structure-of-volatility perspectives both use the concept of *implied volatility*, which is defined as the volatility parameter that justifies a quoted option price. The volatilities implied from options on market indices are now routinely provided, in addition to the levels of the indices themselves, as market indicators. For example, VIX is the volatility implied by the price for options on the S&P 500 Index.

In summary, the use of options contracts can be an important tool for managing investment risk.

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The complete monograph can be found at <http://www.cfapubs.org/toc/rf/2013/2013/3>.

Use your smartphone to scan the QR code to go straight to the webpage.