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The Franchise Value Approach to the Leveraged Company

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The Franchise Value Approach to the Leveraged Company

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Biography

Martin L. Leibowitz is vice chair and chief investment officer at TIAA-CREF, where he is responsible for the overall management of all TIAA-CREF investments. Previously, he was a managing director, director of research, and member of the executive committee with Salomon Brothers. Mr. Leibowitz has authored several books and more than 130 articles, 9 of which have received a *Financial Analysts Journal* Graham and Dodd Award of Excellence. He was the recipient of AIMR's Nicholas Molodovsky Award in 1995 and the James R. Vertin Award in 1998. In 1995, he received the Distinguished Public Service Award from the Public Securities Association and became the first inductee into the Fixed Income Analysts Society's Hall of Fame. Mr. Leibowitz is a trustee of the Carnegie Corporation, the Institute for Advanced Study at Princeton, and the Research Foundation of AIMR. He holds a B.A. and an M.S. from the University of Chicago and a Ph.D. in mathematics from the Courant Institute of New York University.

Acknowledgments

The Franchise Value Approach to the Leveraged Company

This research paper is intended to explain the general development of P/E sensitivity to leverage and, in that sense, to be a companion piece to "The Levered P/E Ratio," published in the November/December 2002 *Financial Analysts Journal* (Leibowitz 2002). "The Levered P/E Ratio" examines how analysts should go about valuing an already-levered company with already-levered return parameters. This research paper also places the leveraged-P/E work in the more general context of the franchise value approach.

Franchise Value

The franchise value approach and its many ramifications for judging corporate value have been examined in a series of research reports and analyses.[1] In particular, Leibowitz and Kogelman (1991) explored the debt problem within the franchise value framework but still took the corporate finance viewpoint of using the return parameters of the unlevered company as the starting point. The basic finding of Leibowitz and Kogelman was that the use of leverage leads to two P/E effects, which tend to offset each other. Moreover, debt can, depending on return parameters of the unlevered company, increase or reduce the P/E. For normal levels of investment-grade debt, however, all of the P/E effects were relatively modest.

This finding may have been theoretically correct from a corporate finance viewpoint—that is, given a company with known return characteristics. But the investment analyst must induce the company's underlying fundamental structure of returns from the return parameters of an already-levered company.

The two most obvious effects of debt are (1) the reduction of earnings because of interest charges and (2) the intrusion of the creditor's claim on the company's assets. These two effects were taken into account in the earlier 1991 franchise value study, but we overlooked an important third effect—how debt changes the appearance of the company's characteristics. It is this third effect that is the focal point of the present paper.

The franchise value technique is a particularly productive framework for exploring the effect of leverage. In this approach, the company is conceptually segmented into two components: (1) a tangible value (TV) that represents the value derived from all past investments and (2) a franchise value (FV) that

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[1]Leibowitz (1998, 2000); Leibowitz and Kogelman (1991, 1994).

incorporates the value associated with all future investments. Because leverage has a very different impact on each of these two components, the FV approach greatly facilitates the analysis of leverage's effect on valuation.

Specifically, the FV decomposition underscores that the current shareholder's participation in the company's growth component depends totally on the excess return on new investment. This excess return can be further parsed into the (1) gross return on assets associated with new investments (or return on investment, ROI) and (2) the capital costs required to fund these future investments.

For an unlevered company, the basic FV structure is that shareholders' value, V', is simply the sum of tangible value and franchise value:

$$V'(r',R') = TV'(r') + FV'(R'),\qquad(1)$$

where $TV'(r')$ is the tangible value from the return on current assets, r', and $FV'(R')$ is the franchise value derived from new investments, R'.

The first step is to apply the Modigliani–Miller theorem (1958a, 1958b) to the first component to obtain a revised tangible value:[2]

$$TV(h|r) = TV'(r') - hA,\qquad(2)$$

where h is the current debt ratio relative to the current assets, A, and r is the after-interest return on the company's current book value.

The next step is to present an argument that the company's franchise value should remain invariant under any future debt policy, $h*$, so that

$$FV(h*|R) = FV'(R'),\qquad(3)$$

where R is the *levered* company's return on equity in new investments.

A key advantage of this approach is that by maintaining the value components of the unlevered company "intact," it allows continued use of the unlevered discounting rate, k', and thereby avoids the issues associated with ascertaining a new risk-adjusted discount rate.

Earlier studies (Leibowitz and Kogelman 1991, 1994) computed the company's equity value, $P(h,h*|r',R')$, in terms of (presumably known)

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[2]Under Modigliani–Miller (1958a), one can argue that future capital costs should depend only on the overall magnitude of the needed capital, not at all on the choice of the equity/debt mix. In a tax-free environment, this result implies that the company's going-forward debt policy will have no impact on either its future return on investment or its capital costs. Consequently, the debt policy should have no impact on the growth component of the company's value. (This assertion ceases to be totally true when taxes and the impact of the tax shield are considered). By shunting aside the question of future debt policy, the argument based on Modigliani–Miller implies that equity valuation is affected only through the level of debt currently in place to support the existing book of business.

characteristics—that is, r', R', and so on—of the underlying unlevered company. In the present study, the challenge is to restate the two value components in terms of the return characteristics of the *leveraged* company to develop an expression that contains only the overt parameters—r, R, and so on—of the leveraged company:

$$P(h, h*|r, R) = P(h, h*|r', R').\tag{4}$$

With the Equation 4 formulation in hand, we can explore how leverage affects the theoretical valuation of a company as the market and analysts perceive it—that is, with a given after-interest return on equity, r, growth rate, g, and earnings retention rate, b. The extension of this approach to the important case of valuation in a taxed environment then becomes relatively straightforward.

Levered Tangible Value

To effect the translation from unlevered to levered parameters, we begin with the *unlevered* earnings, E', before any interest charges. With leverage, the after-interest earnings, E, will be simply

$$\begin{aligned} E &= E' - yhA \\ &= (r' - yh)A, \end{aligned}\tag{5}$$

where y is the corporate interest rate on borrowed capital. The *levered* company's return on equity then becomes

$$\begin{aligned} r &\equiv \frac{E}{B} \\ &= \frac{(r' - yh)A}{(1 - h)A} \\ &= \frac{(r' - yh)}{1 - h}, \end{aligned}\tag{6}$$

and the unlevered return on assets is

$$r' = (1 - h) + yh.\tag{7}$$

The levered tangible value then becomes

$$TV(h|r) = \left(\frac{r'}{k'} - h\right)A$$

$$= \frac{A}{k'}(r' - k'h)$$

$$= \frac{A}{k'}[r(1-h) + yh - k'h] \tag{8}$$

$$= \frac{A}{k'}[r(1-h) - h(k' - y)].$$

Levered Franchise Value

Now, as for the results from future investments, the unlevered FV component can be expressed, for a wide range of conditions, as

$$FV'(R') = \left(\frac{R' - k'}{k'}\right)GA, \tag{9}$$

where GA represents the magnitude of future investment opportunities (in present value terms) on which an *unlevered* return, R', can be earned. At the outset, note that the future return on investment R' can differ from the current return on equity, r'. Every such dollar of opportunity generates a theoretically perpetualized stream of future annual earnings R' that has a discounted present value of R'/k'. By definition, however, a dollar capital investment is required to realize this earnings stream, so the net present value is

$$\frac{R'}{k'} - 1 = \frac{R' - k'}{k'} \tag{10}$$

per dollar of investment opportunity.

To consider the FV component in a leveraged context, first note that leverage $h*$ applied to future investments may be quite different from the debt ratio in the company's current capital structure. The next major assumption is that the magnitude of future investment opportunities, GA, remains invariant. In essence, this assumption is tantamount to presuming that the company is "opportunity constrained" (rather than capital constrained). In other words, even the *unlevered* company is assumed to have access to the equity capital needed to pursue every investment opportunity with a positive net present value. (In today's global capital markets with multiple channels for public, private, and venture capital, this assumption is more reasonable than it would have been in earlier years.) Under this invariance condition, the total gross value generated by all future investments becomes $(R'/k')GA$, regardless of how these opportunities are financed.

Turning now to the cost of financing these future investments, recall that the all-equity route would simply result in the financing cost obtained above. Now, if some level of debt, $h*$, is used, the debt will have a lower financing cost but the increased leverage will raise the cost of future equity. If the Modigliani–Miller theorem is again invoked, the combined cost of financing for all such future debt-plus-equity investments must be based on the underlying risk of the enterprise itself (i.e., it must coincide with the *unlevered* all-equity financing rate, k'. (In the interest of simplicity, the standard practice is adopted of assuming that the same risk-based all-equity rate k' applies to both current and future investment activities.) Thus, because future investments both generate a return and incur a financing cost that remains invariant across all levels of leverage, the franchise value for a given future debt ratio simply equals the unlevered franchise value:

$$FV(h*|R) = FV'(R')$$
$$= \left(\frac{R' - k'}{k'}\right) GA. \tag{11}$$

This invariance result illustrates the convenience of using the FV approach to explore the effects of leverage.

The two value components can now be combined to provide an estimate of the theoretical value $P(h,h*|r,R)$ for the leveraged company:

$$P(h,h*|r,R) = TV(h|r) + FV(h*|R)$$
$$= [TV'(r') - hA] + FV'(R'). \tag{12}$$

The Case of Constant Asset Growth

Recall that the initial objective was to obtain a valuation when given the observed parameters of the already-leveraged company. Equation 9 has already provided the unlevered valuation; now, the task is to find a way to estimate the company's franchise value, $FV'(R')$, in terms of market observables. The two most fundamental factors in a company's future progress are its growth in earnings, g_E, and the retained earnings, bE, required to fund that growth. Our challenge is to find a representation for the franchise value that explicitly incorporates these two parameters.

To move forward, recall our requirement that the company is opportunity-constrained (not capital-constrained). We now further refine this assumption so that all useful investment opportunities grow in perpetuity at a common constant rate g—regardless of whether or not the company uses leverage to fund them. In other words, the total new investment in a given period would be the same for both the *levered* and the *unlevered* company. Leibowitz and

Kogelman (1994) showed that such a growth pattern corresponds to a present value of future investment opportunities that is a multiple, G, of the current book value, where

$$G = \frac{g}{k - g}. \tag{13}$$

Note that g is the growth of the asset base that is available for investment at the new return on investment R'. At the current level of generality, this growth rate need not be the same as either the rate of unlevered earnings growth rate, $g_{E'}$, or the levered earnings growth, g_E.

With this basic growth assumption for the company's assets, the franchise value becomes,

$$FV'(R') = \left(\frac{R' - k'}{k'}\right)\left(\frac{g}{k' - g}\right)A. \tag{14}$$

Of course, Equation 14 is still based on the return characteristics of the unlevered company—r' and R'. To proceed to the next step, we must express these parameters in terms of the corresponding variables for the levered company.

We define the levered company's return on equity in new investments to be

$$R \equiv \frac{\Delta E}{\Delta B}, \tag{15}$$

where ΔB is the change in book value, or $(1 - h*)\Delta A$. Thus,

$$\begin{aligned} \Delta E &= (R' - h*y)\Delta A \\ &= (R' - h*y)\left(\frac{\Delta B}{1 - h*}\right), \end{aligned} \tag{16}$$

so

$$\begin{aligned} R &\equiv \frac{\Delta E}{\Delta B}, \\ &= \frac{R' - h*y}{1 - h*} \end{aligned} \tag{17}$$

and

$$R' = R(1 - h*) + h*y. \tag{18}$$

Equations 17 and 18 for the levered company's franchise value correspond to, respectively, Equation 6 and Equation 7 for the company's current investment base.

Levered Valuation with Differential Returns

The franchise value can now be expressed in terms of the levered parameters:

$$
\begin{aligned}
FV(h^*|R) &= FV'(R') \\
&= \left(\frac{R'-k'}{k'}\right)\left(\frac{g}{k'-g}\right)A \\
&= \left[\frac{R(1-h^*)+h^*y-k'}{k'}\right]\left(\frac{g}{k'-g}\right)A.
\end{aligned}
\tag{19}
$$

At this point, one might question how the going-forward debt policy, h_*, enters the formula after we went to such great lengths to point out that future leveraging should not affect value for a given initially unlevered company. For an answer, remember that the key to the invariance condition is return on new investments, R'. On the one hand, all companies with the same R' and the same growth prospects will have the same franchise value, regardless of current or future levels of debt. On the other hand, *levered* companies that have the same growth rate and the same return on equity, R, may, depending on their debt policies, have very different franchise values. The distinction is that the different debt policies imply different underlying values of R' and hence different franchise values.

It is important to recognize the real nature of this relationship. For a given unlevered return on investment, the addition of leverage does lead to a higher levered return on equity, but a current shareholder's value is based on the excess return from new investments. As noted previously, leveraging does not really change the magnitude of this excess return; hence, the franchise value remains invariant. That is, Equation 11 holds.

When the starting point is a levered company with a given R, however, the unlevered return on investment R' must be induced. The greater the leverage ratio, the lower the underlying R' associated with the given R. Consequently, higher leverage implies lower franchise value and, therefore, lower valuation for the overall company.

In other words, the basic problem is to look through the confounding influence of the debt policy and "find" the franchise value of the underlying company. Once this FV magnitude is found, a changing assumption regarding the future debt level will certainly alter the levered return on equity, but it will

have absolutely no theoretical impact on the unlevered return on investment, the excess return, or the franchise value itself.

Now, the shareholder value formulation can be expressed totally in terms of the levered company:

$$P(h, h^* | r, R) = TV(h | r) + FV(h^* | R)$$

$$= \left[\frac{r(1-h) - h(k'-y)}{k'} \right] A + \left\{ \left[\frac{R(1-h^*) + h^*y - k'}{k'} \right] \left(\frac{g}{k'-g} \right) \right\} A$$

$$= \frac{A}{k'(k'-g)} \left\{ \left[r(1-h) - h(k'-y) \right] (k'-g) + \left[R(1-h^*) + h^*y - k' \right] g \right\}$$

$$= \frac{A}{k'(k'-g)}$$

$$\times \left\{ k' \left[r(1-h) - h(k'-y) \right] + g \left[R(1-h^*) + h^*y - k' - r(1-h) + h(k'-y) \right] \right\}$$
(20)

$$= \frac{A}{k'(k'-g)}$$

$$\times \left\{ k' \left[r(1-h) - h(k'-y) \right] + g \left[-(1-h)k' + R(1-h^*) - r(1-h) + y(h^*-h) \right] \right\}$$

$$= \frac{A}{(k'-g)} \left\{ \left[(1-h)(r-g) - h(k'-y) \right] + \frac{g}{k'} \left[(R-r) - (Rh^* - rh) + (h^*-h)y \right] \right\}.$$

Finally, to obtain the P/E, divide by $E = (1-h)rA$ to obtain the general FV formulation:

$$\frac{P(h, h^* | r, R)}{E} = \left(\frac{1}{k'-g} \right) \frac{1}{r}$$
(21)

$$\times \left\{ \left[(r-g) - \left(\frac{h}{1-h} \right) r_p \right] + \frac{g}{k'(1-h)} [(R-r) - (Rh^* - rh) + (h^*-h)y] \right\},$$

where r_p is the risk premium, defined as $k' - y$.

This generality carries with it certain costs, however, even beyond obvious intractability. For example, in the most general case, the parameters R and g are assumed constant through time, but this assumption implies that the return on existing assets will change over time [i.e., r will converge toward R over time (Leibowitz 1998)]. Similarly, although g represents a constant growth rate of assets, the growth of earnings, g_E, must change each year. By

the same token, for the unlevered company, the general case means that, over time, the value of return on equity r', as well as growth of earnings, will change.

At a given point in time, each of these variables has a well-specified value, so the valuation formulas are valid at that point, which justifies development of this general expression. But these variables—and the associated P/E—will migrate over time, even when the central ongoing parameters—$g, R,$ and $h*$ (or $g, R',$ and $h*$)—are kept fixed.

Multiple Facets of Levered Growth

In the general FV model given in Equation 21, the parameter g represents constant annual growth in investable assets (i.e., growth in the opportunity to earn the excess returns associated with the fixed return on investment, R'). The asset growth can be related to the current return on assets as follows:

$$
\begin{aligned}
g &= \frac{\Delta A}{A} \\
&= \frac{b'E'}{A} \\
&= b'r'.
\end{aligned}
\tag{22}
$$

Note that the retention factor, b', as used here, serves only to scale the incremental annual investment in terms of the earnings level. The actual source of the capital may be total or partial external financing (i.e., it need not literally be reinvested earnings). Moreover, when $r' \neq R'$, then r' will change year by year as more assets are invested at the new fixed rate, R'. The retention factor will also move each year as just enough earnings are "reinvested" to fund constant asset growth g.

In this situation, the earnings growth can be related to the current level of r' as follows:

$$
\begin{aligned}
g_{E'} &= \frac{\Delta E'}{E'} \\
&= \frac{R'\Delta A}{r'A} \\
&= \left(\frac{R'}{r'}\right)g.
\end{aligned}
\tag{23}
$$

Again, keep in mind that r' will trend toward the fixed value of R'. Hence, growth rate, $g_{E'}$, which represents the next year's earnings growth as of a given point in time, will converge toward the fixed rate of asset growth, g.

Moreover, applying Equation 22 to Equation 23 produces

$$g_{E'} = \left(\frac{g}{r'}\right)R'$$

$$= b'R'.$$

(24)

Thus, as might be expected, asset growth g relates to current return on assets r' whereas earnings growth $g_{E'}$ is tied to return on new investments R'. Equations 22 and 24 imply that the two return parameters can always be expressed as the appropriate growth rates divided by retention factor b'. Indeed, all this analytic development could have proceeded by eliminating the return variables and relying only on the retention factor and the two growth rates.

Turning now to the levered company, retain the fundamental assumption of a fixed annual growth rate in investable assets. At the outset, the growth in book value is

$$g_B \equiv \frac{\Delta B}{B}$$

$$= \frac{(1 - h^*)\Delta A}{(1 - h)A}$$

$$= \left(\frac{1 - h^*}{1 - h}\right)g.$$

(25)

With a fixed new debt policy, the level of current debt h will migrate over time toward new debt $h*$. In such situations, the book growth will also be period dependent.

Similarly, for the growth in the after-interest earnings,

$$g_E = \frac{\Delta E}{E}$$

$$= \frac{(R' - h^*y)\Delta A}{(r' - hy)A}$$

$$= \left(\frac{R' - h^*y}{r' - hy}\right)g$$

$$= \left(\frac{R}{r}\right)g,$$

(26)

which is an analogous result to Equation 23. To relate g_E to the earnings retention factor for the levered company requires recognition that b reflects only the equity portion of the incremental investment. That is,

$$\Delta A = bE + h*\Delta A,$$

(27)

so

$$\Delta A = \frac{bE}{1 - h^*}. \tag{28}$$

The growth rate of (fixed) assets can also be expressed as a multiple of this retention rate and the levered return on equity, r:

$$
\begin{aligned}
g &= \frac{\Delta A}{A} \\
&= \frac{bE}{(1 - h^*)A} \\
&= \frac{br(1 - h)A}{(1 - h^*)A} \\
&= br\left(\frac{1 - h}{1 - h^*}\right).
\end{aligned}
\tag{29}
$$

Inserting the relationship given in Equation 29 into Equation 25 yields the following useful result for the book value's growth rate:

$$
\begin{aligned}
g_B &= \left(\frac{1 - h^*}{1 - h}\right) g \\
&= \left(\frac{1 - h^*}{1 - h}\right) br\left(\frac{1 - h}{1 - h^*}\right) \\
&= br.
\end{aligned}
\tag{30}
$$

As a next step, Equation 26 can be combined with Equation 29 and the growth in levered earnings can be expressed in terms of the levered return on investment, as follows:

$$
\begin{aligned}
g_E &= \left(\frac{R}{r}\right) g \\
&= \left(\frac{R}{r}\right)\left[br\left(\frac{1 - h}{1 - h^*}\right)\right] \\
&= bR\left(\frac{1 - h}{1 - h^*}\right).
\end{aligned}
\tag{31}
$$

Again, note that, as in the unlevered situation, the levered return parameters can be eliminated through appropriate use of the retention factor and the growth rates.

Finally, the levered earnings growth (Equation 26) can be tied to the unlevered earnings growth (Equation 23):

$$
\begin{aligned}
g_E &= \left(\frac{R}{r}\right)g \\
&= \left(\frac{R}{r}\right)\left(\frac{r'}{R'}g_E\right) \\
&= \left(\frac{R}{R'}\right)\left(\frac{r'}{r}\right)g_{E'}.
\end{aligned}
\tag{32}
$$

Also, the two retention factors can be related by using Equations 22 and 29:

$$
\begin{aligned}
b' &= \frac{g}{r'} \\
&= \frac{1}{r'}\left[br\left(\frac{1-h}{1-h*}\right)\right] \\
&= b\left(\frac{r}{r'}\right)\left(\frac{1-h}{1-h*}\right).
\end{aligned}
\tag{33}
$$

The Levered Gordon Model

For an unlevered company, the familiar Gordon model expresses a company's value, P (Damodaran 1997), as[3]

$$
P = \left(\frac{1-b'}{k'-g}\right)E',
\tag{34}
$$

where b' is the fraction of earnings that must be retained and reinvested to generate growth g.

This Gordon formula can be rewritten to provide insight into the key drivers of value:

$$
\begin{aligned}
\frac{P}{E} &= \frac{1}{r'}\left[\left(\frac{s'}{k'-g}\right)\right]+1 \\
&= \left(\frac{1}{k'+s'}\right)\left[\left(\frac{s'}{k-g}\right)+1\right],
\end{aligned}
\tag{35}
$$

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[3]For the basic Gordon growth model, see Gordon (1962, 1974).

where s' is the "franchise spread," defined as the spread between the unlevered company's return on assets and the unlevered discount rate—that is, $s' \equiv r' - k'$. This expression underscores the central importance of the franchise spread as the key source of value associated with the company's growth. Without a positive franchise spread, the P/E devolves to a bland $(1/k')$, regardless of how fast the company grows (Leibowitz 2000).

The discussion in preceding sections illustrated the complications that arise when the current values for the return and the debt ratios differ from the future values. Even though the general FV formulation (Equation 21) may be calculated at a point in time, an analyst may have understandable qualms about developing estimates about so many intrinsically uncertain parameters. As an alternative, trying to simplify the basic Gordon growth model itself is certainly a reasonable way to achieve an intuitive framework and a better basis for subjective judgments.

As an interim step toward a more simplified form, the common convention is now adopted that the debt policy remains unchanged over time—that is, $h* = h$—while the generality of R and r is retained. This step leads to the following reduced form for the franchise value P/E:

$$
\begin{aligned}
\frac{P(h,h|r,R)}{E} &= \left(\frac{1}{k'-g}\right)\frac{1}{r}\left[(r-g)-\left(\frac{h}{1-h}\right)r_p + \frac{g}{k'}(R-r)\right] \\
&= \frac{P(h,h|r,r)}{E} + \left(\frac{g}{k'-g}\right)\left(\frac{R-r}{rk'}\right),
\end{aligned}
\tag{36}
$$

and the alternative expression,

$$
\begin{aligned}
\frac{P(h,h|r,R)}{E} &= \frac{P[(0,0)|r,R]}{E} - \left(\frac{1}{k'-g}\right)\left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right) \\
&= \frac{P[(0,0)|r,r]}{E} + \left(\frac{g}{k'-g}\right)\left(\frac{R-r}{rk'}\right) - \left(\frac{1}{k'-g}\right)\left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right).
\end{aligned}
\tag{37}
$$

Equation 37 makes the point that the "connections" between leverage and distinct ROA/ROI values can be viewed as additive terms. Moreover, both terms can have a powerful impact that does not show up in a naive Gordon computation. In particular, an underestimation of the ongoing return on new investments, R, can compensate for P/E overestimation caused by overlooking the leverage effect. Because the current return on assets (based on historical investments), r, is always the more visible parameter and because return on investment R should reflect the best choices among a spectrum of potential new investments, one might expect R to generally exceed r—possibly, by a significant margin. In this case, a naive Gordon P/E based solely

on current ROA could lead to underestimation of the theoretical P/E, whether or not leverage is present.

In addition, a stable debt policy leads to an immediate simplification of the various growth rates. Thus, when $h = h*$, then from Equation 25,

$$g = br$$
$$= g_B;$$

(38)

that is, growth in book value coincides with the constant rate of asset growth. Also, from Equation 31,

$$g_E = bR,$$

(39)

which is now analogous to Equation 24 for the unlevered company. Note that with differential returns, $r' \neq R'$, however, earnings growth rates g_E and $g_{E'}$ will both differ from fixed-asset growth rate g, although they should move toward g as time passes.

To obtain a more tractable form for the levered P/E than Equation 21, the next step is to adopt the (admittedly restrictive) assumption that allows the FV model to devolve into the Gordon format. Basically, what is required is that the ROA and the ROI coincide (that is, R' must equal r') and also, from Equations 7 and 18 (together with the understanding that the debt policy is stable), R is assumed to equal r. This assumption also provides the enormous added benefit that all growth variables will then coincide; that is, from Equations 23 and 32, $g_{E'} = g = g_E$.

Returning now to the levered P/E, by applying the $R = r$ and $h* = h$ conditions to Equation 21, we can finally obtain a tractable and informative levered version of the Gordon model:

$$
\begin{aligned}
\frac{P(h,h|r,r)}{E} &= \left(\frac{1}{k'-g}\right)\left[1 - \frac{g}{r} - \frac{hr_p}{(1-h)r}\right] \\
&= \frac{1 - b - \{hr_p/[(1-h)r]\}}{k'-g} \\
&= \frac{1 - b\{1 + [h/(1-h)](r_p/g)\}}{k'-g} \\
&= \frac{1 - b*}{k'-g},
\end{aligned}
$$

(40)

where $b*$ functions as an effective retention factor—

$$b* = b + \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)$$

$$= b\left[1 + \left(\frac{h}{1-h}\right)\left(\frac{r_p}{g}\right)\right]. \tag{41}$$

The importance of levered P/Es is now clear. The danger in using the naive form of the Gordon model arises in the temptation to improperly combine *levered* retention b with *unlevered* discount rate k', as in

$$\frac{P(0,0|r,r)}{E} = \frac{1-b}{k'-g}. \tag{42}$$

This computation would overstate the theoretical P/E by

$$\frac{\left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)}{k'-g},$$

which becomes quite significant at higher leverage ratios.

Levered "Gordon Components"

Another interesting angle is how the levered Gordon model parses out in terms of the tangible value and the franchise value. With all the Gordon assumptions intact, the tangible value becomes

$$\frac{TV(h|r)}{E} = \left[\frac{A}{k'r(1-h)A}\right][r(1-h) - h(k'-y)]$$

$$= \frac{1}{k'}\left[1 - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)\right] \tag{43}$$

$$= \frac{TV(0|r)}{E}\left[1 - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)\right],$$

where the expression in the last brackets can be viewed as a "connection factor" applied to a naive TV computation. The FV component can be found from

$$\frac{FV(h|r)}{E} = \frac{P(h|r)}{E} - \frac{TV(h|r)}{E}$$

$$= \frac{1 - b - [h/(1-h)](r_p/r)}{k' - g} - \frac{1}{k'}\left[1 - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)\right]$$

$$= \frac{1-b}{k'-g} - \frac{1}{k'} - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)\left(\frac{1}{k'-g} - \frac{1}{k'}\right)$$

$$= \frac{1}{k'(k'-g)}\left\{[(1-b)k' - (k'-g)] - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)[k' - (k'-g)]\right\}$$

$$= \frac{1}{k'(k'-g)}\left[(g - bk') - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)g\right]$$ (44)

$$= \frac{g}{k'(k'-g)}\left[1 - \frac{k'}{r} - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)\right]$$

$$= \left(\frac{g}{k'-g}\right)\left(\frac{1}{k'r}\right)\left[r - k' - \left(\frac{h}{1-h}\right)r_p\right]$$

$$= \frac{FV(0|r)}{E}\left[1 - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r-k'}\right)\right].$$

Thus, one can see that both components of company value that an analyst sees are reduced by the use of debt. The form of the "correction factors" clearly shows, however, that the FV term will be more severely affected on a proportional basis. In particular, companies with high growth rates but modest franchise spreads could have the unfortunate combination of a sizable FV with a significant downward correction factor.[4]

WACC under Gordon Model Assumptions

In using the standard Gordon model to calculate the weighted-average cost of capital (WACC) for levered companies, the proper theoretical procedure is to use the appropriate levered discount rate $k(h)$ that can be applied to after-interest earnings E:

$$\frac{P(h,h|r,r)}{E} = \frac{1-b}{k(h)-g}.$$ (45)

---

[4]Again, recall that a given company with a fixed franchise spread s' (hence, a fixed unlevered return on investment, R') will have an unchanging FV even when it uses debt to fund new investments. For a *levered* company, however, higher levels of debt with a given R imply that its unlevered version will carry a lower franchise spread, a lower return on investment, and thus a lower franchise value.

The problem is that, unless an additional model framework is introduced, $k(h)$ is really not known. Thus, because we have not introduced any risk model, $k(h)$ should perhaps be viewed more as an "effective" levered discount rate than a "risk-adjusted" discount rate.[5]

Nevertheless, it would be helpful to be able to use the results of this analysis to relate this effective discount rate to the levered company's characteristics. This exercise might be problematic for the more complex general case, but for the highly restrictive conditions that led to the simple revised Gordon formula, the effective discount rate can be shown to be the appropriate rate used in the common calculation of WACC (Ross, Westerfield and Jaffe 1988; Taggart 1991; Grinblatt and Titman 1998; Brealey and Myers 2000). To see this connection, the first step is to set

$$\frac{P}{E} = \frac{1 - b - (h/1 - h)(r_p/r)}{k' - g} \tag{46a}$$

and obtain

$$\frac{P}{E}(k' - g) = 1 - b - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right). \tag{46b}$$

Then, using the defining equation for $k(h)$ (Equation 43) produces

$$
\begin{aligned}
k' &= \frac{E}{P}\left[1 - b - \left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right)\right] + g \\
&= \left[\frac{k(h) - g}{1 - b}\right](1 - b) - \left(\frac{E}{P}\right)\left(\frac{h}{1-h}\right)\left(\frac{r_p}{r}\right) + g \\
&= k(h) - \frac{E}{P}\left(\frac{h}{1-h}\right)\frac{r_p}{r} \\
&= k(h) - \frac{[r(1-h)A]hr_p}{P(1-h)r} \\
&= k(h) - \frac{D}{P}(k' - y),
\end{aligned}
\tag{47}
$$

which leads to

$$k'\left(1 + \frac{D}{P}\right) = k(h) + \frac{Dy}{P} \tag{48a}$$

---

[5] Indeed, note that all of the preceding cash flow manipulations were carried out independently of any risk model (including the capital asset pricing model) assumptions.

and

$$k' = \frac{k(h) + y(D/P)}{1 + (D/P)}$$

$$= k(h)\frac{P}{P+D} + y\frac{D}{P+D},$$

(48b)

which is the basic WACC equation. Keep in mind that this discussion demonstrates only that consistency of the (risk-model-free) Gordon growth model with the familiar WACC formulation holds for only the most restrictive Gordon framework.

Another interesting observation is that the simple Gordon model (Equation 30) can be rearranged to provide insight into the nature of the return equilibrium that it represents. First, solve Equation 30 for required return k' as follows:

$$\left(\frac{P}{E}\right)(k' - g) = (1 - b) - \left(\frac{h}{1-h}\right)\left(\frac{k'-y}{r}\right)$$

(49a)

or

$$k'\left[\left(\frac{P}{E}\right) + \left(\frac{h}{1-h}\right)\frac{1}{r}\right] = (1 - b) + g\left(\frac{P}{E}\right) + \left(\frac{h}{1-h}\right)\left(\frac{y}{r}\right).$$

(49b)

And then express k' as

$$k' = \frac{(1-b) + g(P/E) + \{hAy/[r(1-h)A]\}}{P/E + \{hA/[r(1-h)A]\}}$$

$$= \frac{(1-b) + g(P/E) + hy(D/E)}{(P/E) + (D/E)}$$

$$= \frac{(1-b)(E/P) + g + (Dy/E)(E/P)}{1 + (D/E)(E/P)}$$

$$= \frac{(1-b)(E/P) + g + (Dy/P)}{1 + (D/P)}$$

$$= \frac{(1-b)E + gP + Dy}{P+D}.$$

(50)

Equation 50 shows that market return k' is the sum of the dividend, price growth gP, and the interest payments—all divided by enterprise value. In other words, just as one would expect, the totality of the flows generated by the enterprise corresponds to the equilibrium return.

Here, the term "equilibrium" can be taken as signifying that the P/E is stable over time. In the preceding analysis, when this stable (P/E) equilibrium

condition was not met, the percentage change in the P/E had to be present in the numerator. Consequently, $g_{E'}$ would not equal g and would not equal g_E, nor would the simple Gordon formulation (Equation 30) or the WACC formula (Equation 37) hold. This condition underscores the point that all such simple results are valid only under the highly restrictive Gordon assumptions.

Fixed-Earnings-Growth Model

The previous development of the general FV formulation was based on the assumption of a fixed rate of growth for investment opportunities. The earnings growth rate was then treated as a dependent variable. This approach seems to be natural in an opportunity-constrained environment, but a common approach is for the fixed rate of earnings growth to be taken as the starting point. Obviously, when the Gordon assumption that $r = R$ is met, all growth rates coincide and this distinction is irrelevant. When returns are different, however $(r \neq R)$, the selection of a fixed rate of earnings growth does matter, which leads to a different general formulation.

When a fixed earnings growth—as opposed to a fixed growth of investment opportunities—is taken as the starting point, the following formulation can be shown to be the analog to Equation 36 and Equation 37:

$$
\begin{aligned}
\frac{P(h,h|r,T)}{E} &= \left(\frac{1}{k'-g_E}\right)\left\{(1-b)-\left(\frac{h}{1-h}\right)\left[\frac{r_p}{r}-b\left(\frac{R-r}{r}\right)\right]\right\} \\
&= \left\{\frac{P(0,0|r,r)}{E}-\left(\frac{1}{k'-g_E}\right)\left(\frac{h}{1-h}\right)\left(\frac{1}{r}\right)[r_p-b(R-r)]\right\} \qquad (51) \\
&= \left\{\frac{P(0,0|r,r)}{E}-\left(\frac{1}{k'-g_E}\right)\left(\frac{h}{1-h}\right)\left(\frac{1}{r}\right)\left[r_p-\left(g_E-g_A\right)\right]\right\}.
\end{aligned}
$$

Comparing this result with Equation 37 shows that the impact of differential returns is more intertwined with the leverage effect in the fixed-earnings-growth case. Indeed, without leverage, no differential return effect is evident in the P/E. But clearly, where leverage is present and returns are coincident (i.e., $r = R$), both growth assumptions will lead to exactly the same levered Gordon model, Equation 40.

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